Lecture 6: Trigonometric Functions: Final Examples

6.1 Implicit differentiation

Example Suppose we wish to find an equation of the line tangent to the curve

$$8\sin(x) + 2\cos(2y) = 1$$

at $\left(0, \frac{\pi}{6}\right)$. Now

$$\frac{d}{dx}(8\sin(x) + 2\cos(2y)) = \frac{d}{dx}1,$$

so

$$8\cos(x) - 4\sin(2y)\frac{dy}{dx} = 0.$$

Hence

$$4\sin(2y)\frac{dy}{dx} = 8\cos(x).$$

Thus

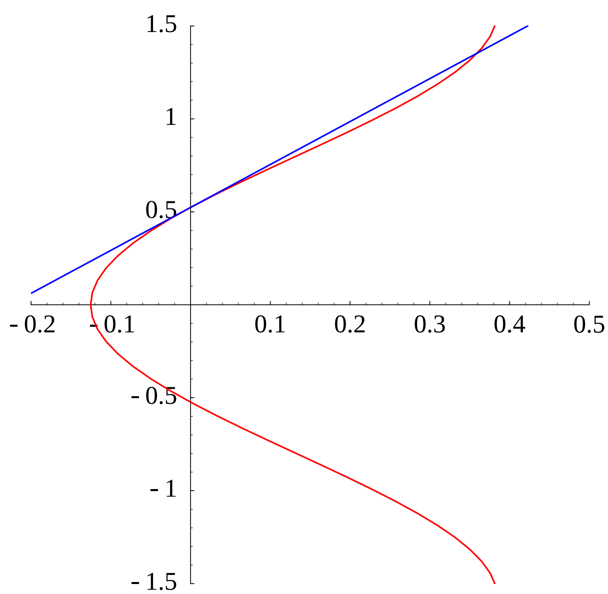
$$\frac{dy}{dx} = \frac{2\cos(x)}{\sin(2y)},$$

and

$$\left.\frac{dy}{dx}\right|_{(x,y)=\left(0,\frac{\pi}{6}\right)} = \frac{2}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{4}{\sqrt{3}}.$$

Hence the equation of the tangent line is

$$y = \frac{4}{\sqrt{3}}x + \frac{\pi}{6}.$$



Graph of $8\sin(x) + 2\cos(2y) = 1$ with tangent line

6.2 Higher order derivatives

Example If $f(x) = \cos(2x)$, then

$$f'(x) = -2\sin(2x),$$

$$f''(x) = -4\cos(2x),$$

$$f'''(x) = 8\sin(2x),$$

and

$$f^{(4)}(x) = 16\cos(2x).$$

From the pattern, we can see that, for example,

$$f^{(10)}(x) = -2^{10}\cos(2x) = -1024\cos(2x).$$

6.3 Linear approximations

Recall: If f is differentiable at a, we call the function

$$L(x) = f(a) + f'(a)(x - a)$$

the *linearization* of f at a. If x is close to a, then

$$f(x) \approx L(x)$$

provides a good approximation of $f(x)$.

Example Let $f(x) = \sin(x)$. Then $f'(x) = \cos(x)$, so $f(0) = 0$ and $f'(0) = 1$. Hence the linearization of f at $x = 0$ is

$$L(x) = x.$$

In other words, for small angles x,

$$\sin(x) \approx x.$$

For example,

$$\sin(0.01) \approx 0.01.$$

Note that the exact value, to 8 decimal places, is

$$\sin(0.01) = 0.00999983.$$