Lecture 6: Types of Functions

6.1 Basic definitions

Definition If $a_0, a_1, \ldots, a_n$ are constants and n is a nonnegative integer, then the function

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

is called a *polynomial*.

Note: The *degree* of a polynomial is the highest power of x in the polynomial with a nonzero coefficient.

Example $f(x) = 3x^2 + 4x - 6$ is a polynomial of degree 2, that is, a quadratic polynomial.

Example $g(t) = 5t - 6$ is a first degree polynomial, sometimes called a *linear* function.

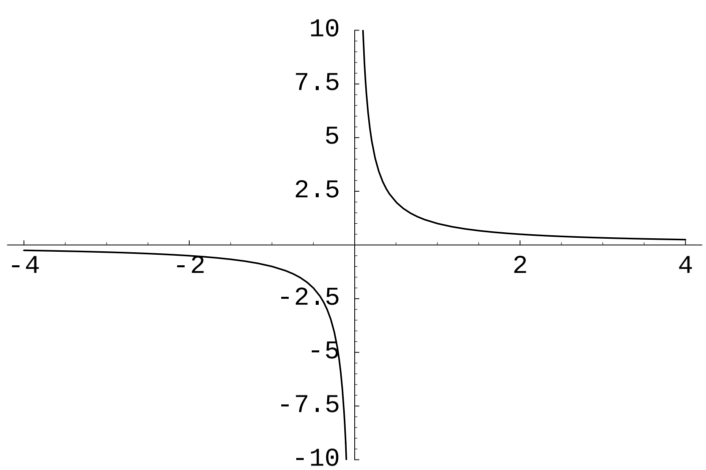
Definition If p and q are both polynomials, then the function

$$f(x) = \frac{p(x)}{q(x)}$$

is called a *rational function*.

Example $f(x) = \dfrac{x^2 - 6}{x + 5}$ is a rational function.

Example $f(x) = \dfrac{1}{x}$ is a rational function. Recall that the graph of f is an example of a *hyperbola* with the x-axis and y-axis for asymptotes.



Example $f(x) = x^2 - \dfrac{6}{x^2} = \dfrac{x^4 - 6}{x^2}$ is a rational function.

Definition Any function which may be built up using the operations of addition, subtraction, multiplication, division, and taking roots is called an *algebraic function*.

Example $f(x) = \sqrt{x}$ is an algebraic function.

Example $f(x) = (x^2 + 2x + 3)^{\frac{2}{3}}$ is an algebraic function.

Note: Every polynomial is a rational function and every rational function is an algebraic function.

Note: For integers p and q, $q \neq 0$, $x^{\frac{p}{q}} = \sqrt[q]{x^p}$. For example, $4^{\frac{3}{2}} = \sqrt{4^3} = \sqrt{64} = 8$.

Definition A function which is not an algebraic function is called a *transcendental function.*

Example $f(x) = \ln(15x + 6)$ is a transcendental function.

Example The trigonometric functions are all transcendental functions.