1. What is the smallest number divisible by three that is the sum of four distinct primes?

(a) 18

(b) 21

(c) 24

(d) 27

(e) None of the above

2. The sum of two numbers is 11. Their difference is 12. What is their product?

(a) 6

(b) $\frac{-11}{2}$

(c) -6

(d) $\frac{-23}{4}$

(e) None of the above

3. Find all non-constant polynomials $p(x)$ so that $p(p(x)) = 2p(x) - 1$.

(a) $p(x) = 2x$

(b) $p(x) = 3x$

(c) $p(x) = 3x + 1$

(d) $p(x) = 2x + 1$

(e) None of the above

4. Find the line of symmetry of the parabola in the form $y = ax^2 + bx + c$ which passes through the points $(-3, 2)$, $(0, 0)$, and $(1, 1)$.

(a) $x = -5$

(b) $x = -.6$

(c) $x = -.7$

(d) $x = -.8$

(e) None of the above

5. How many positive integers less than the base seven number 3212 are divisible by 3?

(a) 375

(b) 376

(c) 377

(d) 378

(e) None of the above

6. Find the polynomial equation with leading coefficient one which has lowest degree among those with $\sqrt{1 + \sqrt{2}}$ as one of its solutions.

(a) $x^2 - 2x - 1 = 0$

(b) $x^4 - 2x - 1 = 0$

(c) $x^4 - 2x^2 - 1 = 0$

(d) $x^4 - 4x^2 - 1 = 0$

(e) None of the above

7. How many squares can be traced on the following grid?
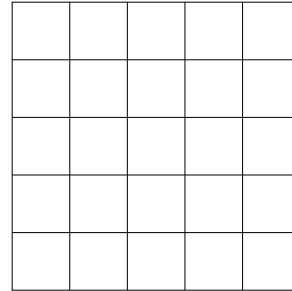
(a) 50

(b) 55

(c) 60

(d) 65

(e) None of the above

8. For what range of real numbers b would the parabola $y = x^2 + bx + 2$ have no points in common with the line $y = 1$? Give the largest range possible.

(a) $-1 < b < 1$

(b) $-.5 < b < .5$

(c) $-1.5 < b < 1.5$

(d) $-2 < b < 2$

(e) None of the above

9. If a, b, and c are positive, and if $ab = 2$, $bc = 3$, and $ac = 6$, what is $a + b + c$?

(a) 5

(b) 6

(c) 12

(d) 14

(e) None of the above

10. Find the sum of all numbers x which are twice as far from 0 on the number line as they are from the number 3.

(a) 2

(b) 6

(c) 7

(d) 12

(e) None of the above

11. What is the largest integer x so that x^3 divides 40,000?

   (a) 10   (b) 20

   (c) 25   (d) 26

   (e) None of the above

12. How many different necklaces can be made by stringing 4 red beads and 2 blue beads together?

   (a) 3   (b) 4

   (c) 5   (d) 6

   (e) None of the above

13. If $f(x-2) = x^2 + 1$, which of the following is equal to $f(x+1)$?

   (a) $f(x) = x^2 + 7x + 6$ (b) $f(x) = x^2 + 7x + 8$

   (c) $f(x) = x^2 + 5x + 9$ (d) $f(x) = x^2 + 6x + 10$

   (e) None of the above

14. A worm crawls up a 53 foot pole 3 feet each day, but slips back 2 feet each night. After how many days will he reach the top?

   (a) 51   (b) 52

   (c) 53   (d) 54

   (e) None of the above

15. A group of people buy larks together (on a lark.) If each person paid 9 Franc, there would be 11 Franc left over after the purchase. If, however, each person contributed only 6 Franc, there would be a shortfall of 16 Franc. How many people are in the group?

   (a) 10   (b) 11

   (c) 12   (d) 13

   (e) None of the above

16. What is the largest number of regions that the plane can be divided into by using 100 straight lines?

   (a) 5050   (b) 5051

   (c) 5052   (d) 5053

   (e) None of the above

17. 28982 is a palindromic number because it reads the same forwards as it does backwards. What is the sum of the digits of the next largest palindromic number?

   (a) 20   (b) 22

   (c) 24   (d) 26

   (e) None of the above

18. A tournament between Ute and Jute ends when one of the two wins two games in a row. How many ways can the tournament turn out if Ute wins no more than two games?

   (a) 6   (b) 7

   (c) 8   (d) 9

   (e) None of the above

19. One root of $x^4 + x^3 + x^2 - x - 2$ is -1. Find the sum of all other real roots.

   (a) 1   (b) 3

   (c) 5   (d) 7

   (e) None of the above

20. The product of three consecutive positive integers is 21 times their sum. Find the sum of the three integers.

   (a) 24   (b) 27

   (c) 30   (d) 33

   (e) None of the above

21. A square flat piece of metal was used to construct a 2-inch high open box. This was accomplished by cutting 2-inch by 2-inch squares from each corner and folding up the sides. The resulting box holds 18 cubic inches. How wide was the original square?

(a)   6 inches

(b) 7 inches

(c)   8 inches

(d) 9 inches

(e) None of the above

22. For what value of a is the polynomial

$$x^3 + 2x^2y + axy^2 + 2y^3$$

divisible by $x + y$?

(a)   0

(b) 1

(c)   2

(d) 3

(e) None of the above

23. When Darby remarked that the apples seemed very small this week, the seller offered to throw an extra apple into the dollar basket. Darby noted that this reduced the price *per dozen* by 5 cents and she then purchased the basket. How many apples did she get for her dollar?

(a)   15

(b) 16

(c)   17

(d) 18

(e) None of the above

24. A tree was planted when it was 4 feet tall and thereafter grew an equal number of feet each year. At the end of 6 years it was twice as tall as it had been at the end of 2 years. How tall was the tree at the end of 4 years?

(a)   12 feet

(b) 13 feet

(c)   14 feet

(d) 15 feet

(e) None of the above

25. Find the largest x so that $|x| < \frac{1}{2}$, and

$$x^4 + x^3 - 3x^2 + x = 0.$$

(a)   0

(b) $\frac{\sqrt{2}-1}{4}$

(c) $\frac{\sqrt{2}-1}{2}$

(d) 1

(e) None of the above

26. Suppose the positive integers are written in succession

$$12345678910111213141516\ldots.$$

What digit appears in the thousandth place?

(a)   0

(b) 1

(c)   2

(d) 3

(e) None of the above

27. The roots of $x^3 - 11x^2 - x + 2 = 0$ are $r, s,$ and t. What is the value of $r^2st + rs^2t + rst^2$?

(a) -11

(b) -22

(c) -33

(d) -44

(e) None of the above

28. In the figure $ABCD$ is a square of side 2 and E, F, G, and H are the midpoints of the sides. What is the area of square $PQRS$?
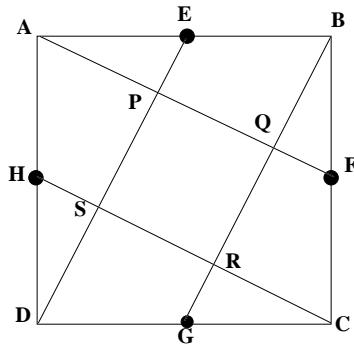
(a) $\frac{3}{4}$

(b) $\frac{4}{5}$

(c) $\frac{5}{6}$

(d) $\frac{6}{7}$

(e) None of the above

29. A radiator holds 16 liters of an antifreeze-water mixture that is 30% antifreeze. Your job is to drain just the right amount of fluid from the radiator so that when the radiator is refilled with pure antifreeze, the mixture becomes 50% antifreeze. How many liters do you need to drain?

(a) $\frac{29}{7}$

(b) $\frac{30}{7}$

(c) $\frac{31}{7}$

(d) $\frac{32}{7}$

(e) None of the above

30. You are being evaluated in three different categories. In category I you can receive either a 0, 1, or 2. In categories II and III you can receive either a 0, 1, 2, 3, or 4. Your ultimate rating is the sum of the points you receive in each category. In how many different ways could you end up with a rating of 5?

(a) 10

(b) 11

(c) 12

(d) 13

(e) None of the above

31. At the thrift store, Hannah got a real bargain – a programmable scientific calculator that never overflows. The only catch is that the display only shows the one's digit. If Hannah put in a 7 and pressed the square button 2000 times, what did the display read at the end?

(a) 1

(b) 9

(c) 3

(d) 7

(e) None of the above

32. Madison's dog chewed on her homework before she could finish it. The fragment saved from the horrible canine's mouth reveal only the two terms of highest degree of the polynomial $p(x)$. It looked like

$$p(x) = x^{18} - 3x^{17} + \ldots.$$

Madison found roots $1, 2, 3, \ldots 17$. What is the missing root?

(a) 18

(b) 17

(c) -18

(d) -151

(e) None of the above

Bonus Questions: Show all your work. The solution to number 33 should be written on the green sheet labeled "41", and the solution to number 34 should be written on the red sheet labeled "42."

33. An 8×11 piece of paper is folded by bringing together opposite vertices. Find the length of the crease.

34. The sum of a certain number of positive integers is 20. Find, with proof, the largest that their product can be.