1. How many integers between 500 and 1500 are perfect squares?

(1) 16   (2) 17

(3) 18   (4) 19

(5) None of the above

2. If $x - y = xy = 1 - x - y$, what is $x + y$?

(1) $\frac{1}{2}$ (2) $\frac{5}{6}$

(3) $\frac{7}{8}$ (4) $\frac{3}{4}$
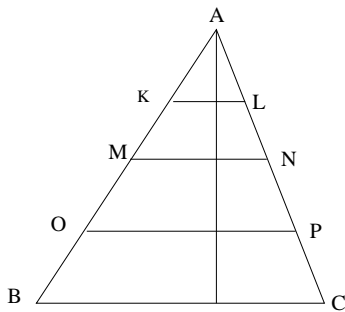
(5) None of the above

3. Triangle ABC has area 80.

$$AK = KM = MO = OB$$

and

$$AL = LN = NP = PC.$$

Find the area of quadrilateral $MNPO$.

(1) 20   (2) 25

(3) 28   (4) 30

(5) None of the above

4. The expression 32 in base b represents the same number as does 21 in base c, while the expression 21 in base b represents the same number as does 13 in base c. What is b?

(1) 3   (2) 5

(3) 7   (4) 8

(5) None of the above

5. Ben has won 20% more chess matches than Amy, while Amy has won 25% more chess matches than Mary Kay. By what percent does the number that Ben has won exceed the number that Mary Kay has won?

(1) 40%   (2) 45%

(3) 50%   (4) 55%

(5) None of the above

6. How many digits are in the number obtained by computing the product

$$(5,123,456) \times (4,134,567) \times (44,311,207)?$$

(1) 17   (2) 18

(3) 19   (4) 20

(5) None of the above

7. One laser blast will break asteroids larger than 10 kg. into three pieces, each with one third the mass of the original. Asteroids smaller than 10 kg. would be shattered into harmless dust. How many laser blasts would be required to reduce a 1000 kg. asteroid to dust?

(1) 364   (2) 365

(3) 366   (4) 367

(5) None of the above

8. A forester seeds a tract of land under the assumption that one mature tree will be produced for every S seeds sown. The goal is to raise T mature trees per square mile. Unfortunately on this particular tract there are C chipmunks per square mile, and they consume all the seeds. How many pounds of seeds are consumed per chipmunk, if P seeds weighs one pound?

(1) $\frac{CP}{TS}$ (2) $\frac{CPT}{S}$

(3) $\frac{S}{CTP}$ (4) $\frac{TS}{CP}$

(5) None of the above

9. Let P be the point $(10,0)$. How long is the line segment $\overline{PQ}$ if $\overline{PQ}$ is tangent at Q to the circle $x^2 + y^2 = 1$?

(1) $3\sqrt{10}$ (2) $3\sqrt{11}$

(3) $2\sqrt{10}$ (4) $2\sqrt{11}$

(5) None of the above

10. What is the area of the triangle with vertices at $(7, 3)$, $(85, 2)$, and $(13, 3)$?

(1) 3 (2) $\frac{10}{3}$

(3) $\frac{11}{3}$ (4) 4

(5) None of the above

11. All x so that $\sqrt{2 - x} = 1 + \sqrt{x}$ can be written in the form $a + b\sqrt{3}$, where a and b are rational numbers. For all such x as above, find the sum of all such a's and b's.

(1) $-\frac{1}{2}$ (2) $\frac{1}{2}$

(3) 2 (4) $\frac{3}{2}$

(5) None of the above

12. In what number base does $62 \times 14 = 808$?

(1) 12 (2) 14

(3) 16 (4) 18

(5) None of the above

13. Darby enjoys a game called "High-Low" in which she tries to guess an integer chosen between 1 and 1000. After each guess, she is told if her guess is "too high" or "too low," or "correct." How many guesses must Darby be given in order always to be able to guess the number? (You may assume that Darby is plenty clever.)

(1) 9 (2) 10

(3) 11 (4) 12

(5) None of the above

14. If 12 carpenters can construct 3 homes in 20 days, how many days will it take 10 carpenters to construct 4 homes?

(1) 29 (2) 30

(3) 31 (4) 32

(5) None of the above

15. How many pairs (x, y) of positive integers can be found to solve

$$2x + 7y = 1000?$$

(1) 71 (2) 72

(3) 73 (4) 74

(5) None of the above

16. What is the volume of a cube which is inscribed in a sphere of radius 1?

(1) 1 (2) $\sqrt{3}$

(3) $\frac{8\sqrt{3}}{9}$ (4) $\frac{2\sqrt{3}}{3}$

(5) None of the above

17. You are unexpectedly left in charge of the Mitchell Road Presbyterian Church Nursery. There is a playing area covered by a rug, a large empty playpen, and ten cribs, each containing one screaming infant. You are given these instructions by the Nursery Coordinator – right before she hopped a plane to Bermuda:

1. Repeat instructions 2) through 7) until all cribs are empty.

2. Remove one infant from a crib and place it in the large playpen.

3. Repeat instructions 4) through 6) until the playpen is empty.

4. Lift one infant from the playpen.

5. If his or her diaper is wet, then change it.

6. Set this infant on the rug.

7. Return all infants on the rug to the large playpen.

What is the maximum number of diapers you might have to change if you follow these instructions?

(1) 10 (2) 25

(3) 50 (4) 55

(5) None of the above

18. In triangle ABC, D is a point of side $\overline{BC}$. If $AC = 3$, $AD = 3$, $BD = 8$, and $CD = 1$, find AB.

(1) 8
(2) 9
(3) 10
(4) 11
(5) None of the above

19. An elastic string, laying along the interval $[-2, 2]$ on the $x-$axis, is stretched uniformly and displaced so that it then lays along $[3, 9]$. What is the new location of the point of the string which formerly was at $x = 1$?

(1) 6
(2) 6.5
(3) 7
(4) 7.5
(5) None of the above

20. Find the sum of the squares of the digits of the smallest prime factor of 104,060,401.

(1) 2
(2) 12
(3) 23
(4) 90
(5) None of the above

21. By definition, the *harmonic mean* of two numbers is the reciprocal of the average of the reciprocals of the numbers. The harmonic mean of two numbers is 4. One of the numbers is 3. What is the other number?

(1) 2
(2) 4
(3) 6
(4) 8
(5) None of the above

22. Among the dogs owners at the Furman University Kennel Club dog show are Mr. Basset, Miss Beagle, Mr. Shepherd and Mr. Spitz, each of whom is the namesake of a breed of dog brought by one of the other three. In an altercation with the shepherd, Mr. Spitz' dog bit the shepherd's owner's wife. The basset's owner kept his dog well away from the scuffle. Who owns the spitz?

(1) Mr. Shepherd
(2) Miss Beagle
(3) Mr. Basset
(4) Mr. Spitz
(5) None of the above

23. What is the sum of the squares of the roots of $x^4 - 5x^2 + 6 = 0$?

(1) 9
(2) 10
(3) 11
(4) 12
(5) None of the above

24. Ryan was ill and had to take the calculus test a day late. His 96 was found to raise the class average on the test from 71 to 72. How many students, including Ryan, took the test?

(1) 22
(2) 24
(3) 26
(4) 28
(5) None of the above

25. From a point P on the circumference of a circle, perpendiculars $\overline{PA}$ and $\overline{PB}$ are dropped to points A and B on two mutually perpendicular diameters. If $AB = 10$, find the length of a diameter.

(1) 15
(2) 16
(3) 18
(4) 20
(5) None of the above

26. Suppose that A and B are integers so that

$$\sqrt{9 + 6\sqrt{2}} = \sqrt{A} + \sqrt{B}.$$

What is $A + B$?

(1) 9
(2) 10
(3) 11
(4) 12
(5) None of the above

27. If s men working s hours a day complete a job in s days, how many days would the job take t men working t hours per day?

(1) $\frac{s}{t}$
(2) $\frac{s^3}{t^2}$
(3) $\frac{t^2}{s^3}$
(4) $\frac{s^2}{t}$
(5) None of the above

28. Find the area of the region consisting of all points (x, y) so that

$$1 \le |x| + |y| \le 2.$$

(1) 4
(2) $\pi + 1$
(3) 5
(4) 6
(5) None of the above

29. For how many integers N is

$$N^4 + 6N < 6N^3 + N^2?$$

(1) 1   (2) 2

(3) 3   (4) 4

(5) None of the above

30. Assuming that two teams are evenly matched (each has probability $\frac{1}{2}$ of winning any game), what is the probability that the World Series will require the full seven games? (The series goes on until one team has won four games.)

(1) $\frac{1}{2}$ (2) $\frac{2}{3}$

(3) $\frac{5}{16}$ (4) $\frac{9}{32}$

(5) None of the above

31. If $A = 2^{35}$, $B = 5^{15}$ and $C = 6^{14}$, arrange these in increasing order.

(1) ABC (2) BCA

(3) CAB (4) ACB

(5) None of the above

32. Note the identity

$$\frac{1}{x(x+1)(x+2)} = \frac{1}{2}\left(\frac{1}{x} - \frac{2}{x+1} + \frac{1}{x+2}\right).$$

Using this identity, find the sum

$$\frac{1}{1\cdot 2\cdot 3} + \frac{1}{2\cdot 3\cdot 4} + \frac{1}{3\cdot 4\cdot 5} + \cdots + \frac{1}{98\cdot 99\cdot 100}.$$

(1) $\frac{4950}{19800}$ (2) $\frac{4951}{19800}$

(3) $\frac{4952}{19800}$ (4) $\frac{4953}{19800}$

(5) None of the above

Bonus Questions: Show all your work.
The solution to No. 1 should be written on the yellow sheet labeled "41", and the solution to No. 2 should be written on the blue sheet labeled "42." These should be available from your proctor.

1. Evaluate

$$3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \dots}}}.$$

Show your work.

2. Cylindrical oil drums one meter in diameter are being brought into a 200 meter by 200 meter storage yard and packed together as shown. Approximate the number of barrels that will fit in the yard.