1. Every birthday of my life, my mother has seen to it that my cake contains my age in candles. Starting on my fourth birthday, I have always blown out all my candles. Before that age, I averaged a 50% total blowout rate. So far, I have blown out exactly 900 candles. How old am I?

(1) 45 (2) 44

(3) 43 (4) 42

(5) None of the above

2. Two sentries start at point B. Sentry 1 walks back and forth between points A and B, taking 28 seconds to make the complete trip. Sentry 2 walks back and forth between points B and C, taking 90 seconds for the trip. Both are unwavering in their pace. How many seconds after they start will the two first meet back at point B again? (In seconds).

(1) 1000 (2) 1120

(3) 1260 (4) 2520

(5) None of the above

3. How many integers from 1 to 8000 have no factors (other than 1) in common with 8000?

(1) 3200 (2) 2400

(3) 2000 (4) 2800

(5) None of the above

4. A line divides a circle of radius 2 into two arcs. If the length of the smaller arc is $\pi/3$, what is the area of the region bounded by the smaller arc and the line?

(1) $\frac{\pi}{3} - 1$ (2) $\frac{\pi}{3} - \frac{1}{4}$

(3) $\frac{\pi}{3} - \frac{2}{3}$ (4) $\frac{\pi}{3} - \frac{1}{2}$

(5) None of the above

5. If the base 9 number $8888888877777777\cdots 11111111$ is divided by 8, what is the remainder?

(1) 3 (2) 2

(3) 0 (4) 1

(5) None of the above

6. A parallelogram has 3 of its vertices at $(1,2)$, $(3,8)$, and $(4,1)$. What is the sum of all of the possible first coordinates for the other vertex?

(1) 6 (2) 9

(3) 8 (4) 7

(5) None of the above

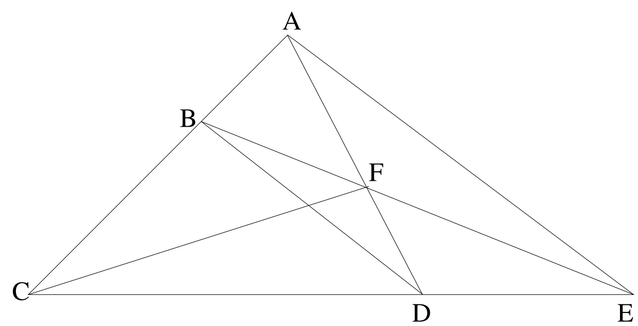
7. Let S be the set of all 5-digit numbers that can be formed from the set $\{1,2,3,4,5,6,7,8,9\}$ by selecting digits from the set without replacement. Let N be the subset of S consisting of all those elements of S for which it is true that the digits 2, 4, and 6 appear, and are in the proper order. For example, 12346 is an element of N, as is 75246. How many elements does N contain?

(1) 300 (2) 600

(3) 180 (4) 450

(5) None of the above

8. In the accompanying figure, triangle ACE is given with $\overline{BD}$ parallel to $\overline{AE}$ and point F is the intersection of segments $\overline{BE}$ and $\overline{AD}$.



Consider the following statements:

1. $\triangle BFA$ is similar to $\triangle DFE$
2. $\triangle AFE$ is similar to $\triangle DFB$
3. $\triangle ACE$ is similar to $\triangle BCD$
4. $\triangle BFC$ is similar to $\triangle DCF$

How many of the above statements are true?

(1) 0 (2) 1

(3) 2 (4) 3

(5) None of the above

9. What is the degree measure of the acute angle θ which satisfies the equation

$$\cos(81^\circ) + \cos(39^\circ) = \cos(\theta)?$$

(1) 18 (2) 19
(3) 21 (4) 20
(5) None of the above

10. The cardinality of

$$\{x \mid e^{-x \log_e 5} = 25\}$$

is what integer?

(1) 0 (2) 1
(3) 2 (4) 3
(5) None of the above

11. The average age of a group of mathematicians and computer scientists is 40. If the mathematicians' average age is 35 and the computer scientists' average age is 50, what is the ratio of the number of mathematicians to the number of computer scientists?

(1) 2.5 (2) 3.5
(3) 2 (4) 3
(5) None of the above

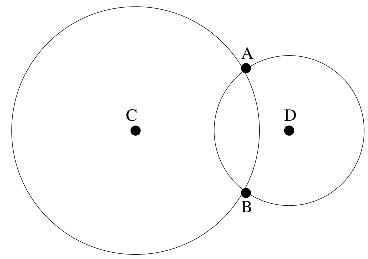
12. Let $f(x)$ equal

$$x^{2x} + x^{1+2x} + x^{2+2x} + \dots.$$

What is $f(1/2)$?

(1) 2 (2) 1/2
(3) 3/2 (4) 1
(5) None of the above

13. In the accompanying figure, two circles with centers C and D intersect at points A and B. If the line segment $\overline{AB}$ intersects $\overline{CD}$ at E so that $EC = 2ED$, then:



(1) the measure of $\angle ADB$ is twice the measure of $\angle ACB$
(2) the ratio of the area of the larger circle to the smaller circle is 4 to 1
(3) the area of triangle ABC is twice the area of triangle ABD
(4) $AC = 2AD$
(5) None of the above

14. How many 9's are in the decimal expansion of the number 999999899999^2? Note that this is the square of a 12-digit number.

(1) 13 (2) 12
(3) 11 (4) 10
(5) None of the above

15. How many distinct triangles with positive integer side lengths have perimeter equal to 50?

(1) 48 (2) 50
(3) 51 (4) 49
(5) None of the above

16. What is a if

$$a = \log(\tan(1^\circ)) + \log(\tan(2^\circ)) + \cdots + \log(\tan(89^\circ))$$

where the logarithm is a base 10 logarithm?

(1) 15 (2) 5
(3) 0 (4) 10
(5) None of the above

17. Suppose that g is which a function which satisfies

$$xg(2009 - x) + g(x) = x.$$

What is $g(2008)$?

(1) 4016 (2) 0

(3) 1 (4) 2008

(5) None of the above

18. Erin forgot to write down the last homework problem. Hannah told her that they had to factor a certain 5th degree polynomial that Ms. Smith said had integer roots. Unfortunately, the cell phone connection was bad, and Erin only understood the beginning and end of the polynomial. Thus she knew it had the form

$$P(x) = x^5 - 10x^4 + ? + 14.$$

Erin was able to factor it into irreducible linear factors. How many distinct linear factors did it have?

(1) 2 (2) 1

(3) 4 (4) 3

(5) None of the above

19. Walter's bird feeder sits atop a rigid pole of height 100 inches that is planted in soft ground. A 10-mph wind tilts the pole, spilling birdseed on the ground 4 inches from the base of the pole. If the pole's angle from the vertical is proportional to the wind speed, how far from the base of the pole does the bird seed land when there is a 20-mph wind?

(1) $\frac{16\sqrt{39}}{25}$ (2) $\frac{16\sqrt{41}}{25}$

(3) $\frac{32\sqrt{39}}{25}$ (4) $\frac{32\sqrt{41}}{25}$

(5) None of the above

20. What is the sum of all of the digits of all of the integers from 1 to 1,000,000?

(1) 27,000,001 (2) 26,000,998

(3) 27,000,000 (4) 26,000,999

(5) None of the above

21. A rectangle has its vertices at the points $(0, 0)$, $(7, 0)$, $(7, 4)$, and $(0, 4)$. Assume the sides of the rectangle are mirrors and a laser is shining from vertex $(0, 0)$ in such a way that the beam starts out going up 2 units for each 5 units to the right it travels. Which vertex will the beam next reach?

(1) $(7, 4)$ (2) $(7, 0)$

(3) $(0, 4)$ (4) $(0, 0)$

(5) None of the above

22. For how many values of a does the system below have exactly 3 solutions?

$$\begin{cases} x^2 - y^2 = 0 \\ (x - a)^2 + y^2 = 1 \end{cases}$$

(1) 1 (2) infinitely many

(3) 0 (4) 2

(5) None of the above

23. If

$$\log_2(\log_3(\log_4(x)) = 0$$

and

$$\log_3(\log_4(\log_2 y)) = 0$$

and

$$\log_4(\log_2(\log_3 z)) = 0$$

then the sum of x, y, and z is:

(1) 89 (2) 58

(3) 105 (4) 50

(5) None of the above

24. A truncated right circular cone has an 8 cm radius for its lower base, a 6 cm radius for its upper base, and a 6 cm height. What is its volume, in cubic centimeters?

(1) $\frac{1064\pi}{7}$ (2) $\frac{294\pi}{7}$

(3) $\frac{2072\pi}{7}$ (4) $\frac{6216\pi}{7}$

(5) None of the above

25. A cylindrical pot with a diameter of 10 inches has water to a depth of 1 inch. When a smaller cylindrical pot of height 10 inches is placed inside (with its bottom flush to the bottom of the larger pot), the water level rises 2 inches to a new height of 3 inches. What is the smaller pot's diameter?

(1) $10\sqrt{1/3}$ (2) $10\sqrt{2/5}$

(3) $10\sqrt{1/4}$ (4) $10\sqrt{2/3}$

(5) None of the above

26. Express

$$2(10^3 e^{2\pi i} + \frac{1}{2}e^{\pi i})$$

as an integer.

(1) 2002 (2) 2000

(3) 2003 (4) 2001

(5) None of the above

27. How many polynomials p of degree three or less satisfy

$$p(x-y) = p(x) + p(y)$$

for all x and y?

(1) 2 (2) 0

(3) Infinitely many (4) 1

(5) None of the above

28. There are 6 lanes on the circular track at the Furman field house. Tom runs in the middle of the inside lane and finishes one mile (8 laps) in 7 minutes. Mark runs in the middle of the outside lane and finishes a mile (7 laps) in 8 minutes. What is the distance, in feet, from the middle of the inside lane to the middle of the outside lane? (Hint: There are 5280 feet in one mile.)

(1) $\frac{5280}{112\pi}$ (2) $\frac{5280}{116\pi}$

(3) $\frac{5280}{114\pi}$ (4) $\frac{5280}{110\pi}$

(5) None of the above

29. A ball dropped from 8 feet takes 8/10 of a second to land. On each successive bounce the time to drop is 80% of the preceding drop. How long until the ball stops bouncing?

(1) 6.4 seconds (2) 4 seconds

(3) 8.8 seconds (4) 7.2 seconds

(5) None of the above

30. A partition of an integer n is a set of positive integers $\{k_1, k_2, \dots, k_s\}$ with $k_1 \leq k_2 \leq \cdots \leq k_s$ so that $n = k_1 + k_2 + \cdots k_s$. For example, there are three partitions of 3: $\{3\}$, $\{1, 2\}$ and $\{1, 1, 1\}$. How many partitions of 9 are there?

(1) 30 (2) 31

(3) 29 (4) 32

(5) None of the above

31. How many 10-digit positive integers use each and every one of the ten digits 0, 1, $\cdots$, 9?

(1) 3265620 (2) 3628800

(3) 3265920 (4) 3123490

(5) None of the above

32. How many of the numbers in the previous problem are prime?

(1) 100 (2) 1000

(3) 10000 (4) 100000

(5) None of the above

Bonus Questions: Show all your work – use the colored sheets provided by the proctors

1. Suppose that you are going to connect the points $(4, 4)$, $(x, 3)$, $(x, 0)$, and $(-8, -4)$ with straight line segments, obtaining a path consisting of the union of three line segments. What is the shortest such path you can make? Explain.

2. Let c_n be an arithmetic sequence which begins $c_1 = 8$, $c_2 = 10$, $c_3 = 12$. Let b_n be defined by $b_1 = 6$ and $b_n = b_{n-1} + c_n$. Write an explicit expression for b_n in terms of n.