

1. In Furmantopia, there are only coins worth 3 funits or 5 funits. What is the price (in funits) of the most expensive item one can't pay for using these coins, if exact change is required?

(1) 11 (2) 17
 (3) 7 (4) 22
 (5) None of the above

2. A truncated right circular cone has an 8 cm radius for its lower base, a 6 cm radius for its upper base, and a 6 cm height. What is its volume, in cubic centimeters?

(1) $\frac{1064\pi}{7}$ (2) $\frac{294\pi}{7}$
 (3) $\frac{2072\pi}{7}$ (4) $\frac{6216\pi}{7}$
 (5) None of the above

3. For how many values of a does the system below have exactly 3 solutions?

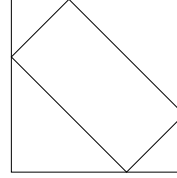
$$\begin{cases} x^2 - y^2 = 0 \\ (x - a)^2 + y^2 = 1 \end{cases}$$

(1) 1 (2) infinitely many
 (3) 0 (4) 2
 (5) None of the above

4. An unbroken trolley track two miles long expands uniformly a length of two feet, causing the track to buckle in the middle and rise so as to form an isosceles triangle. The distance of the vertex of the triangle above the ground is

(1) between 100 and 1000 feet
 (2) between 10 and 100 feet
 (3) between one and two feet
 (4) less than one foot
 (5) None of the above

5. A rectangle with unequal sides is placed in a square so that each vertex lies on a side of the square at a trisection point of that side. Find the fraction of the area of the square that is covered by the rectangle.



(1) $5/9$ (2) $1/2$
 (3) $4/9$ (4) $1/3$
 (5) None of the above

6. In a town designed by an eccentric geometer, there are only ten roads. Each is straight but exactly two are parallel and exactly two roads meet at every intersection. How many intersections are there in the town?

(1) 45 (2) 43
 (3) 42 (4) 44
 (5) None of the above

7. Every birthday of my life, my mother has seen to it that my cake contains my age in candles. Starting on my fourth birthday, I have always blown out all my candles. Before that age, I averaged a 50% total blowout rate. So far, I have blown out exactly 900 candles. How old am I?

(1) 45 (2) 44
 (3) 43 (4) 42
 (5) None of the above

8. Two sentries start at point B . Sentry 1 walks back and forth between points A and B , taking 28 seconds to make the complete trip. Sentry 2 walks back and forth between points B and C , taking 90 seconds for the trip. Both are unwavering in their pace. How many seconds after they start will the two first meet back at point B again? (In seconds).

(1) 1000 (2) 1120
 (3) 1260 (4) 2520
 (5) None of the above

9. What is the sum of all of the digits of all of the integers from 1 to 1,000,000?

(1) 27,000,001 (2) 26,000,998
 (3) 27,000,000 (4) 26,000,999
 (5) None of the above

10. Suppose that $\alpha = \sqrt{2}$, and that

$$S = \sqrt{1 + \frac{1}{\alpha} \sqrt{1 + \frac{1}{\alpha} \sqrt{\dots}}}$$

What is S ?

(1) $\sqrt{3}/2$ (2) $\sqrt{2}$
 (3) $\sqrt{2}/2$ (4) $\sqrt{3}$
 (5) None of the above

11. Hannah, Paige, and Darby each took the same 7-question True/False quiz. Hannah and Paige each got 6 questions right. Their answers were:

Hannah – FFTTTTF

Paige – TFFTTTF

When Darby took the same quiz, her answers were:

Darby – TTFFTTT

What is the most you can say about how many Darby got right?

(1) Exactly 3
 (2) Exactly 4
 (3) At least 3
 (4) This can't be determined
 (5) None of the above

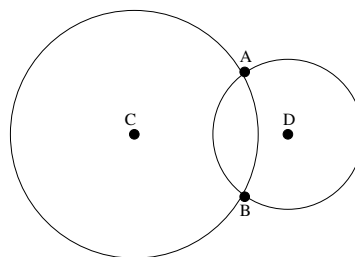
12. A box contains 10 red marbles, 11 blue marbles, and 12 green marbles. You reach into the box and pull some out, hoping to get at least 3 of the same color. The smallest number of marbles you can pull out and be sure to accomplish this is:

(1) 7 (2) 4
 (3) 24 (4) 22
 (5) None of the above

13. The average age of a group of mathematicians and computer scientists is 40. If the mathematicians' average age is 35 and the computer scientists' average age is 50, what is the ratio of the number of mathematicians to the number of computer scientists?

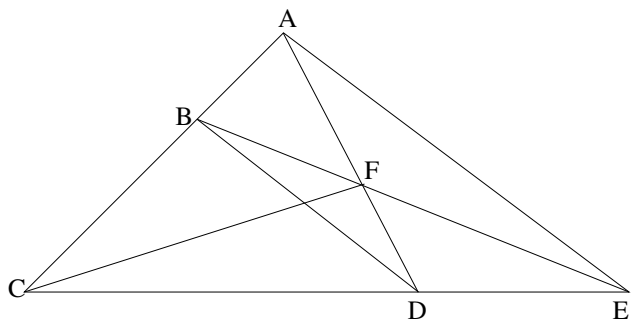
(1) 2.5 (2) 3.5
 (3) 2 (4) 3
 (5) None of the above

14. In the accompanying figure, two circles with centers C and D intersect at points A and B . If the line segment \overline{AB} intersects \overline{CD} at E so that $EC = 2ED$, then:



- (1) the measure of $\angle ADB$ is twice the measure of $\angle ACB$
 (2) the ratio of the area of the larger circle to the smaller circle is 4 to 1
 (3) the area of triangle ABC is twice the area of triangle ABD
 (4) $AC = 2AD$
 (5) None of the above

15. In the accompanying figure, triangle ACE is given with \overline{BD} parallel to \overline{AE} and point F is the intersection of segments \overline{BE} and \overline{AD} .



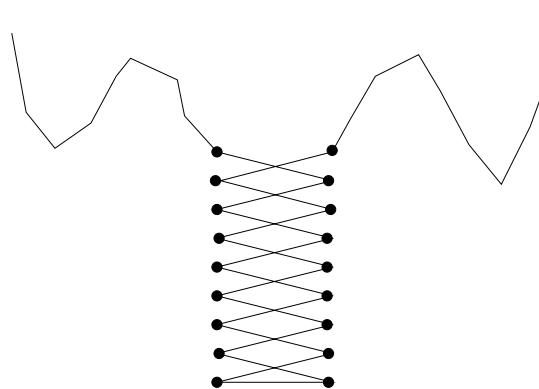
Consider the following statements:

1. $\triangle BFA$ is similar to $\triangle DFE$
2. $\triangle AFE$ is similar to $\triangle DFB$
3. $\triangle ACE$ is similar to $\triangle BCD$
4. $\triangle BFC$ is similar to $\triangle DCF$

How many of the above statements are true?

- (1) 0
 - (2) 1
 - (3) 2
 - (4) 3
 - (5) None of the above
16. How many 3-digit natural numbers are there which are multiples of 7?
- (1) 125
 - (2) 142
 - (3) 128
 - (4) 144
 - (5) None of the above

17. Kevin's FleetFeet special running shoes have 9 eyelets on each side, spaced $1/2$ inches apart. Using a standard lacing (see figure below), Kevin laces his right shoe so that there is one inch between the two parallel rows of eyelets. This leaves 12 inches of lace free on each side for tying. How long is the lace?



- (1) $24 + 8\sqrt{5}$
 - (2) $13 + 8\sqrt{5}$
 - (3) $25 + 7\sqrt{5}$
 - (4) $25 + 8\sqrt{5}$
 - (5) None of the above
18. If the base 9 number $8888888877777777 \dots 11111111$ is divided by 8, what is the remainder?
- (1) 3
 - (2) 2
 - (3) 0
 - (4) 1
 - (5) None of the above
19. Darby got all five of the daily problems right one day. This raised her average from .200 to .240. How many daily problems had she been given prior to that day?
- (1) 95
 - (2) 100
 - (3) 85
 - (4) 80
 - (5) None of the above

20. The Furman bell tower only has two bells; one sound a “ding” and the other a “dong”. The board of trustees has passed a rule that a ding cannot be the second note after a ding, and a dong cannot be the third note after a ding. If a new freshman music major is told to start a tune with “ding, ding”, what is the longest tune he can legally play?

- (1) ten notes
- (2) he can play forever
- (3) six notes
- (4) seven notes
- (5) None of the above

21. Ben always insists on cutting his own birthday cake. Although he cuts straight, he cuts rather randomly. Although any two cuts always intersect, no three cuts intersect at a common point. All in all Ben made six cuts. How many pieces did he end up with?

- (1) 22
- (2) 20
- (3) 12
- (4) 24
- (5) None of the above

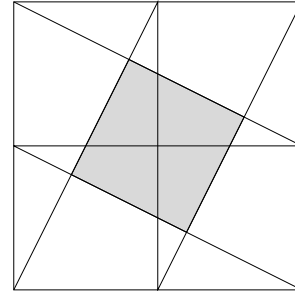
22. How many 10-digit positive integers use each and every one of the ten digits 0, 1, \dots , 9?

- (1) 3265620
- (2) 3628800
- (3) 3265920
- (4) 3123490
- (5) None of the above

23. How many of the numbers in the previous problem are prime?

- (1) 100
- (2) 1000
- (3) 10000
- (4) 100000
- (5) None of the above

24. What is the area of the shaded square shown if the sidelength of the larger square is 1?

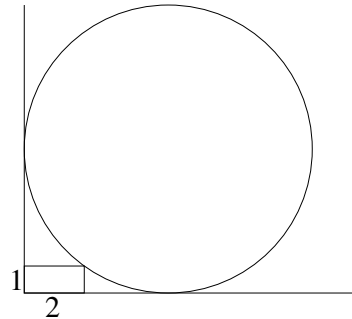


- (1) $1/5$
- (2) $1/3$
- (3) $1/4$
- (4) $1/2\sqrt{3}$
- (5) None of the above

25. In some non-base-10 number system, $3 + 4 = 10$ and $3 \cdot 4 = 15$. What is 3^4 ?

- (1) 143
- (2) 141
- (3) 144
- (4) 142
- (5) None of the above

26. What is the radius of the circle below?



- (1) 5.5
- (2) 4
- (3) 4.5
- (4) 5
- (5) None of the above

27. How many polynomials p of degree three or less satisfy

$$p(x - y) = p(x) + p(y)$$

for all x and y ?

- (1) 2 (2) 0
 (3) Infinitely many (4) 1
 (5) None of the above

28. There are 6 lanes on the circular track at the Furman field house. Tom runs in the middle of the inside lane and finishes one mile (8 laps) in 7 minutes. Mark runs in the middle of the outside lane and finishes a mile (7 laps) in 8 minutes. What is the distance, in feet, from the middle of the inside lane to the middle of the outside lane? (Hint: There are 5280 feet in one mile.)

- (1) $\frac{5280}{112\pi}$ (2) $\frac{5280}{116\pi}$
 (3) $\frac{5280}{114\pi}$ (4) $\frac{5280}{110\pi}$
 (5) None of the above

29. A parallelogram has 3 of its vertices at $(1, 2)$, $(3, 8)$, and $(4, 1)$. What is the sum of all of the possible first coordinates for the other vertex?

- (1) 6 (2) 9
 (3) 8 (4) 7
 (5) None of the above

30. A cylindrical pot with a diameter of 10 inches has water to a depth of 1 inch. When a smaller cylindrical pot of height 10 inches is placed inside (with its bottom flush to the bottom of the larger pot), the water level rises 2 inches to a new height of 3 inches. What is the smaller pot's diameter?

- (1) $10\sqrt{1/3}$ (2) $10\sqrt{2/5}$
 (3) $10\sqrt{1/4}$ (4) $10\sqrt{2/3}$
 (5) None of the above

31. What digit does 2008^{2008} end in?

- (1) 6 (2) 2
 (3) 0 (4) 4
 (5) None of the above

32. How many integers from 1 to 8000 have no factors (other than 1) in common with 8000?

- (1) 3200 (2) 2400
 (3) 2000 (4) 2800
 (5) None of the above

Bonus Questions: Show all your work – use the colored sheets provided by the proctors.

- Suppose that you are going to connect the points $(4, 4)$, $(x, 3)$, $(x, 0)$, and $(-8, -4)$ with straight line segments, obtaining a path consisting of the union of three line segments. What is the shortest such path you can make? Explain.
- Let c_n be an arithmetic sequence which begins $c_1 = 8$, $c_2 = 10$, $c_3 = 12$. Let b_n be defined by $b_1 = 6$ and $b_n = b_{n-1} + c_n$. Write an explicit expression for b_n in terms of n .