Furman University
Wylie Mathematics Tournament
Ciphering Competition

March 8, 2008

House Rules

1. All answers are positive integers(!)
2. All answers must be written in standard form. For example, 8 not 2^3, and 10, not $\binom{5}{2}$.

Division II Round I Ciphering

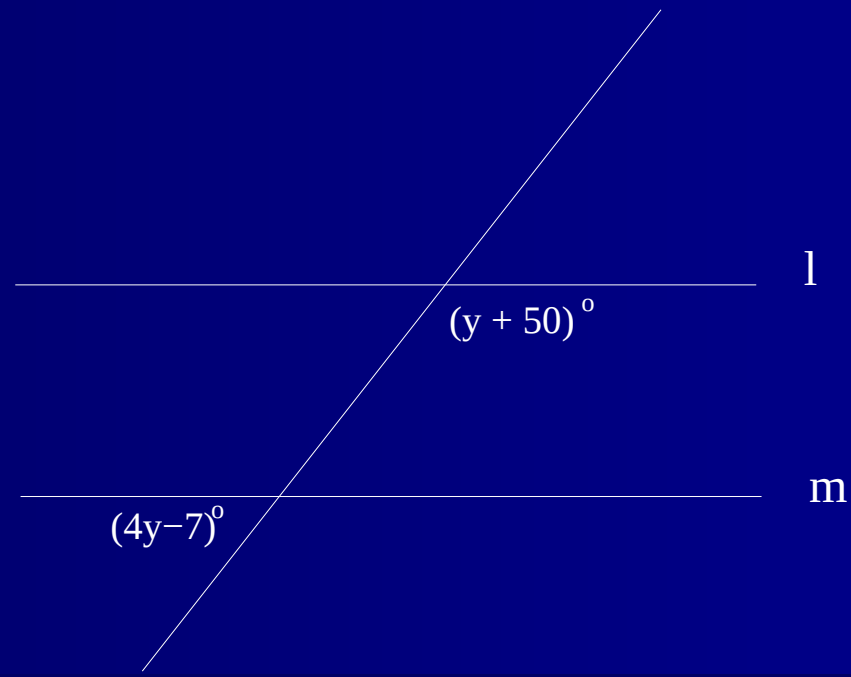
Participants in Round I ciphering from Division II schools should now make their way to the front.

Division II Round I – Number 1

Jim is considering what sandwich to buy for lunch. He has a choice of 2 different types of bread, 3 different types of cheese, and 4 different types of meat. For any sandwich, he must choose one type of bread, and either one type of cheese or one type of meat, or one type of cheese and one type of meat. How many different sandwiches can he choose?

Division II Round I – Number 2

In the figure below, lines l and m are parallel. What is the value of $5y$?



Division II Round I – Number 3

In a league that has 5 teams, each team plays each of the other teams twice in a season. What is the total number of games played during the season?

Division II Round I – Number 4

The surface area of a sphere is 3600π square inches. If V represents the volume of the sphere in cubic inches, what is $\frac{V}{\pi}$?

Division II Round I – Number 5

Let (a, b) be the vertex of the graph of $f(x) = 2x^2 - 4x + 5$. What is $a + b$?

Division II Round I – Number 6

The function $f(x) = x^3 + 1$ is one-to-one; therefore the inverse function $y = f^{-1}(x)$ exists. What is $f^{-1}(9)$?

Division II Round I – Number 7

A standard six-sided die is weighted so that the probability of throwing a 1 is twice the probability of throwing a 2. Also, the probability of throwing any particular number greater than 2 is equal to the probability of throwing a 2. The die is to be thrown twice. Let p be the probability that the sum of the numbers thrown will be 4. What is $49p$?

Division II Round II Ciphering

Participants in Round II ciphering from Division II schools should now make their way to the front.

Division II Round II – Number 1

There is a class of 40 girls. There are 18 girls who like to play chess, and 23 who like to play soccer. Several of them like biking. The number of girls who like to play both chess and soccer is 9. There are 7 girls who like chess and biking, and 12 who like soccer and biking. There are 4 girls who like all three activities. In addition, we know that every one of the 40 girls likes at least one of these activities. How many girls like biking?

Division II Round II – Number 2

A staircase has 5 steps. You can walk up the staircase by taking one or two steps at a time. How many different ways can you walk up the staircase?

Division II Round II – Number 3

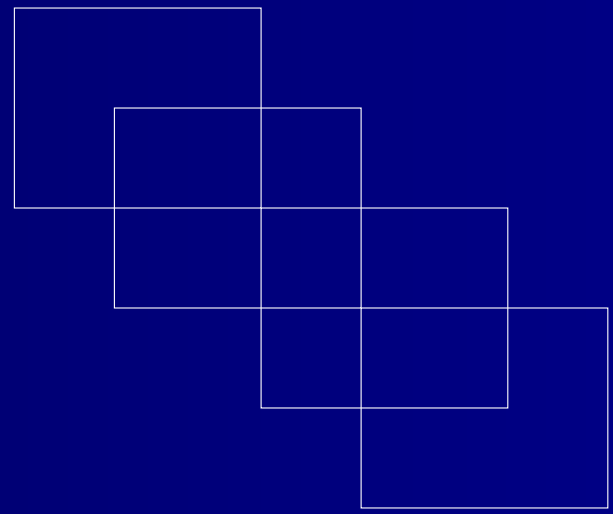
A sixty-page newspaper, which consists of only one section, has only the sheet with page 7 missing. What is the sum of the page numbers that are missing from the paper?

Division II Round II – Number 4

How many 1’s are in the binary representation of 73?

Division II Round II – Number 5

How many rectangles of any size and shape are in the figure below?



Division II Round II – Number 6

Find $-\cot^2(\frac{\pi}{6}) + \csc^2(\frac{\pi}{6})$.

Division II Round II – Number 7

What is the product of the roots of the polynomial $x^3 + 5x^2 - 4x - 20$?

Division II Round III Ciphering

Participants in Round III ciphering from Division II schools should now make their way to the front.

Division II Round III – Number 1

What is the final digit of 7^{2008}?

Division II Round III – Number 2

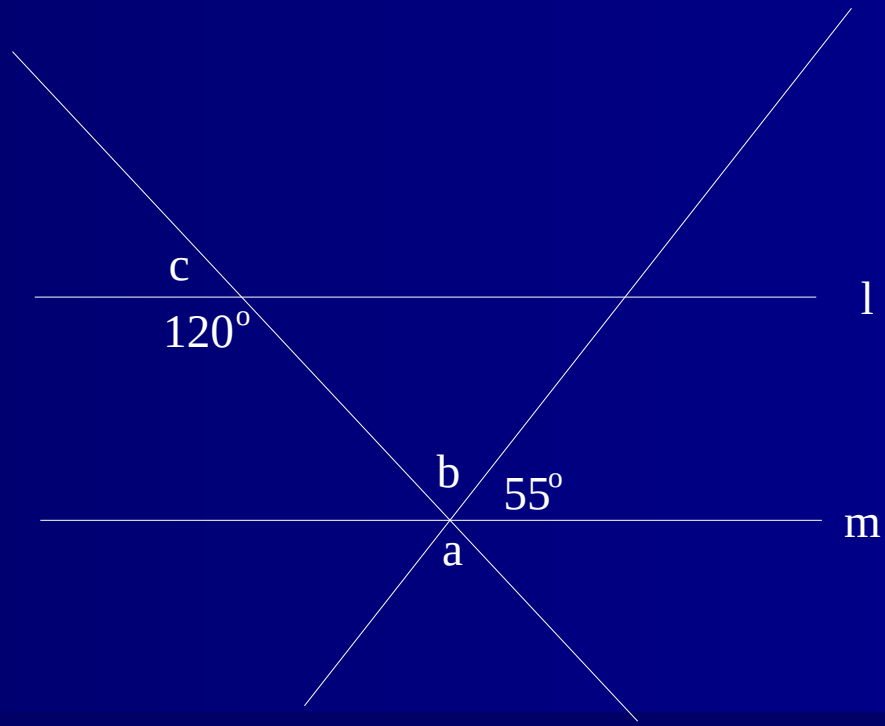
An isoceles triangle, ABC has a base of length 24 and two sides of length 13. What other base measure can an isoceles triangle DEF have, if DEF has equal sides of length 13 and has the same area as triangle ABC?

Division II Round III – Number 3

Find the largest positive integer x such that the distance between $(x, 3, 4)$ and $(7, 3, -4)$ is 10.

Division II Round III – Number 4

Use the sketch below to find the measure of $a+b+c$ (in degrees). In the sketch, lines l and m are parallel.



Division II Round III – Number 5

The complement of an angle measures 10° less than one-fifth of its supplement. Find the measure of the angle, in degrees.

Division II Round III – Number 6

What is $1 \cdot 2 + 3 \cdot 4 + \cdots + 19 \cdot 20$?

Division II Round III – Number 7

For his art class, Bryce bought 2 kg of dark clay and 3 kg of light clay, paying $22 for the clay. He later needed 1 kg of dark clay and 2 kg of light clay, costing $13 altogether. How much more per kg does the dark clay cost than the light clay, in dollars?

Division I Round I Ciphering

Participants in Round I ciphering from Division I schools should now make their way to the front.

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Division I Round I – Number 1

What is the coefficient of the term involving x^3y^4 in the expansion of $(4x - 3y^2)^5$?

Division I Round I – Number 2

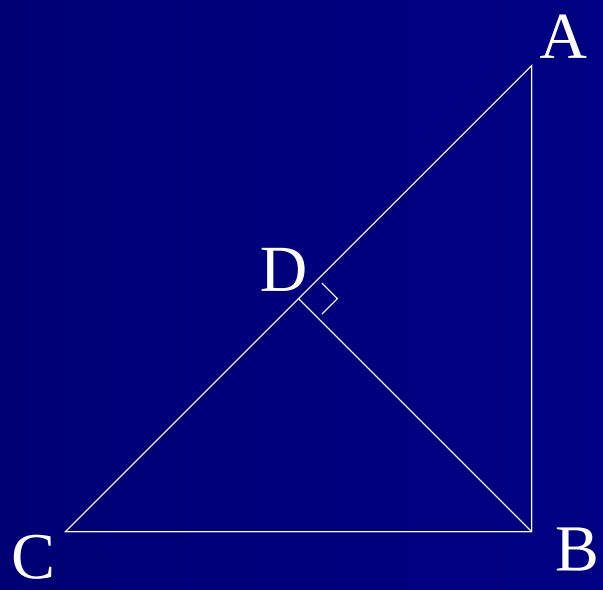
What is the area in the second quadrant bounded by the lines $-4x + 4y = 24$ and $-x + y = 2$?

Division I Round I – Number 3

Suppose that $14!$ is written as $2^p 3^q 5^r 7^s 11^t 13^u$. What is $p + q + r + s + t + u$?

Division I Round I – Number 4

In the figure ABC is a right triangle with right angle at B. $\overline{BD}$ is perpendicular to $\overline{AC}$. If $CD = 9$ and $AD = 3$, then what is $BC + AB + BD$ to the nearest integer?



Division I Round I – Number 5

If $\tan(\theta) = 5$ find

$$\frac{62}{\cot(\theta) + \cos^2(\theta)}$$

Division I Round I – Number 6

The mathematician Augustus DeMorgan lived in the nineteenth century. The over-40-year-old mathematician made the following statement: “I was x years old in the year x^2." In what year was he born?

Division I Round I – Number 7

Chipper and Dull collected thirty-two acorns on Monday and stored them with their initial acorn supply. After Chipper fell asleep, Dull ate half of the acorns. This pattern (gathering 32 and Dull eating half) continued through Friday night. On Saturday morning, Chipper counted the acorns and found that they had 35 remaining. How many acorns did Chipper and Dull have initially stored before they gathered acorns on Monday?

Division I Round II Ciphering

Participants in Round II ciphering from Division I schools should now make their way to the front.

Division I Round II – Number 1

If $g(x) = \frac{x-7}{4x}$ and $f(g(x)) = -x$, then what is $-8g(2) + 4f(2)$?

Division I Round II – Number 2

The perimeter of the isoceles triangle ABC (with $AB = BC$) is 128 inches. The altitude BD is 48 inches. What is the area of the triangle ABC, in square inches?

Division I Round II – Number 3

If a, b, and c are *digits* for which

$$(7a2) - (48b) = (c73),$$

what is $a + b + c$?

Division I Round II – Number 4

Let the foci of the hyperbola $\frac{y^2}{9} - \frac{x^2}{36} = 1$ be (x_1, y_1) and (x_2, y_2). What is $-y_1 \cdot y_2$?

Division I Round II – Number 5

Evaluate $-12\tan(\cos^{-1}(-\frac{12}{13}))$.

Division I Round II – Number 6

At his birthday party, Mr. Green would not directly tell how old he was. He said, “If you add the year of my birth to this year, subtract the year of my tenth birthday and subtract the year of my fiftieth birthday, then add my present age, the result is 80.” How old is Mr. Green?

Division I Round II – Number 7

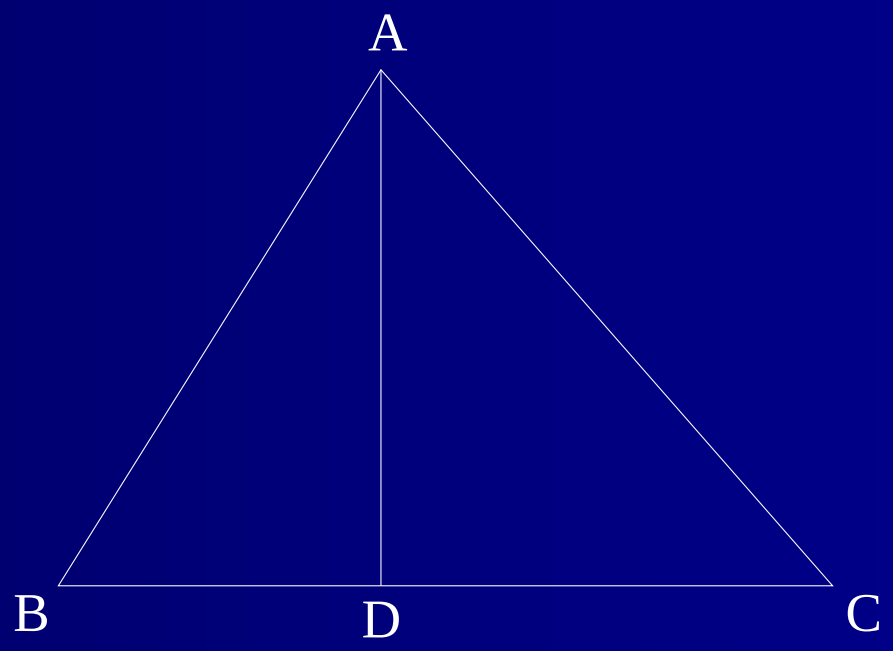
A man buys some shirts and some ties. The shirts cost $7 each and the ties cost $3 each. If the man spends exactly $81 and buys the maximum number of shirts possible under these conditions, what is the total number of items he bought?

Division I Round III Ciphering

Participants in Round III ciphering from Division I schools should now make their way to the front.

Division I Round III – Number 1

In the diagram below, $AB = 5$, $BD = 2$, $DC = 4$, and $CA = 7$. Find AD using the law of cosines.



Division I Round III – Number 2

If Paul can paint a fence in 2 hours and Fred can paint the same fence in 3 hours. How many minutes will it take Paul and Fred working together to paint the fence?

Division I Round III – Number 3

Pat and Chris have the same birthday. Pat is twice as old as Chris was when Pat was as old as Chris is now. If Pat is now 24 years old, how old is Chris?

Division I Round III – Number 4

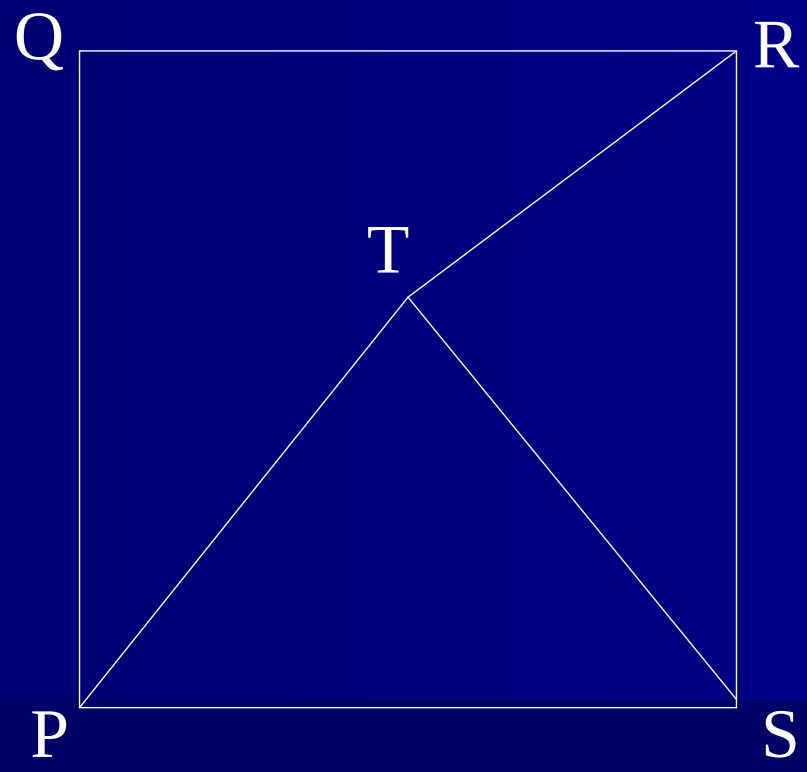
How many numbers between 100 and 400 begin or end with the digit 2?

Division I Round III – Number 5

A circle is inscribed in a given square and another circle is circumscribed about the same square. If the area of the circumscribed circle is 4, what is the area of the inscribed circle?

Division I Round III – Number 6

$PQRS$ is a square and PTS is an equilateral triangle. How many degrees are there in the angle TRS?



Division I Round III – Number 7

Suppose that $\sin(\alpha) = \frac{4}{5}$ and $\sin(\beta) = \frac{5}{13}$, where $0 < \alpha < \frac{\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$. Find $-65\cos(\alpha + \beta)$.

That’s All, Folks
Awards Ceremony to follow soon. Please be patient while we tally the results.