1. You must form a committee, and you determine that:

1. If Aaron is on the committee, then Barbara wants to be on it.
2. Darron won't serve if Charles is on the committee.
3. You feel that Erin should be on the committee.
4. Aaron and Charles agree that at least one of them should serve.
5. Frank wants to be included only if Barbara and Charles take part.
6. Darron wants to be included if and only if Erin is.

Put these people in alphabetic order and assign them their cardinal numbers (so that Aaron is assigned one and Barbara is assigned two, etc.) What is the sum of the numbers assigned to the people who end up on the committee, if everyone is satisfied?

(1) 9 (2) 10
(3) 11 (4) ⋆12
(5) None of the above

2. An illegible receipt shows 72 canned hams were purchased for $x67.9y$. What is $x+y$?

(1) 2 (2) ⋆5
(3) 6 (4) 9
(5) None of the above

3. Four old cannonballs, each touching the other three, are placed pyramid–like next to a cannon down by the VFW. If each cannonball has radius 10 cm, how high is the top of the topmost cannonball above the ground?

(1) $\star 20+20\sqrt{2/3}$ (2) $20+20\sqrt{3}$
(3) $20+10\sqrt{3}$ (4) $20+10\sqrt{2}$
(5) None of the above

4. What is the area of the pentagon whose vertices are $(0,4),(3,0),(6,1),(7,5)$, and $(4,9)$?

(1) 36 (2) ⋆36.5
(3) 37 (4) 37.5
(5) None of the above

5. If $A=.7$, $B=(1/3)^{(1/3)}$, and $C=(1/2)^{(1/2)}$. Put A, B, and C in increasing order.

(1) ⋆BAC (2) ABC
(3) BCA (4) CBA
(5) None of the above

6. For positive real numbers define "weirdo addition" by $x\oplus y=xy$, and "weirdo multiplication" by $x\otimes y=y^x$. For example, $2\oplus 3=6$ and $2\otimes 3=9$. What is the sum of the numbers of the following laws which are valid for all real numbers x and y and z?

1. $x\otimes y=y\otimes x$
2. $x\otimes(y\oplus z)=(x\otimes y)\oplus(x\otimes z)$
3. $(y\oplus z)\otimes x=(y\otimes x)\oplus(z\otimes x)$
4. $(x\otimes y)\otimes z=x\otimes(y\otimes z)$
5. $x\oplus(y\oplus z)=(x\oplus y)\oplus z$

(1) 6 (2) ⋆7
(3) 8 (4) 9
(5) None of the above

7. Find the area of the planar region defined by

$$1\le |x|+|y| \text{ and } x^2-2x+1\le 1-y^2.$$

(1) $\star(3/4)\pi$ (2) $\pi/2$
(3) π (4) $\pi/4$
(5) None of the above

8. Hannah and Darby each tell the truth 3/4 of the time. If Darby makes a statement to Hannah, and Hannah, knowing the truth or falsity of Darby's statement, reports that Darby told the truth, what is the probability that Darby did tell the truth?

(1) 3/4 (2) 4/5
(3) 5/6 (4) ⋆9/10
(5) None of the above

9. If x is the largest root of $(\log_{10} x)^2 - 4\log_{10} x = 7$, what is true about $x/100$?

(1) It is between 1000 and 2000

(2) ⋆It is between 2000 and 3000

(3) It is between 3000 and 4000

(4) It is bigger than 4000

(5) None of the above

10. The polynomial $x^2 - 1088x + 295680$ has two positive integral roots whose greatest common divisor is 16. Find the least common multiple of the two roots.

(1) 18,240 (2) ⋆18,480

(3) 18,960 (4) 19,240

(5) None of the above

11. There are 10 chairs in a row at the speaker's table, and the seat assignments are determined randomly. What is the probability that the vice–president and the treasurer (who hate each other) will have to sit next to each other?

(1) 1/3 (2) 1/4

(3) ⋆1/5 (4) 1/6

(5) None of the above

12. What is the sum of the digits of the smallest positive integer n such that $n/2$ is a perfect square and $n/3$ is a perfect cube?

(1) ⋆18 (2) 21

(3) 24 (4) 27

(5) None of the above

13. An algorithm generates a sequence a_1, a_2,, a_3, ... of approximations to the quantity $\sqrt{7}$. The absolute errors satisfy

$$|a_n - \sqrt{7}| < \frac{1}{(n+3)^2}.$$

Based on this information, how large should n be to guarantee an error less than .000001?

(1) 994 (2) ⋆997

(3) 1003 (4) 1006

(5) None of the above

14. Kevin has sticks of length 1 and length 4. If he cuts the longer stick into two pieces at a random location, what is the probability that he will be able to form a triangle with his three pieces?

(1) 1/2 (2) 1/3

(3) ⋆1/4 (4) 1/5

(5) None of the above

15. Given the following set of points in the plane: $(0,0), (0,1), (0,2), (1,0), (1,1), (1,2), (2,0), (2,1), (2,2)$, how many triangles can be formed with vertices chosen from this set?

(1) 72 (2) 74

(3) ⋆76 (4) 84

(5) None of the above

16. The sum of all products which comprise the multiplication table of the numbers 1 through 2 is 9. What is the sum of all products which comprise a standard multiplication table of the numbers 1 through 20?

(1) ⋆44,100 (2) 44,200

(3) 44,300 (4) 44,400

(5) None of the above

17. How many integers from 1 to 1001 are divisible by either 2, 3, or 5?

(1) 701 (2) ⋆734

(3) 801 (4) 834

(5) None of the above

18. By counting very quickly you notice that your car tire makes 672 revolutions in one mile. What is the radius of the tire rounded to the nearest inch?

(1) 14 (2) ⋆15
(3) 16 (4) 17
(5) None of the above

19. The nine numbers 6561, 2187, 729, 243, 81, 27, 9, 3, and 1 are put into a 3×3 grid so that each number appears exactly once in the grid. They are positioned in such a way that the product of any row or any column is the same. What must this product be?

(1) 534141 (2) ⋆531441
(3) 531423 (4) 421423
(5) None of the above

20. The positive integers are arranged to form a really big expression like this:

$$1234567891011121314151617\ldots.$$

What is the number in the 2007th position?

(1) 6 (2) 7
(3) 8 (4) 9
(5) None of the above

21. How many real solutions are there to the equation

$$e^x - e^{-x} = 5?$$

(1) 0 (2) ⋆1
(3) 2 (4) 3
(5) None of the above

22. Let R be the range of the function $e^{\tan^{-1} x}$. Which of these are true about R? (In all of the following, a and b represent positive real numbers.)

(1) ⋆It has the form (a, b).
(2) It has the form (a, ∞).
(3) It has the form $[a, b]$.
(4) It has the form $[a, \infty)$.
(5) None of the above

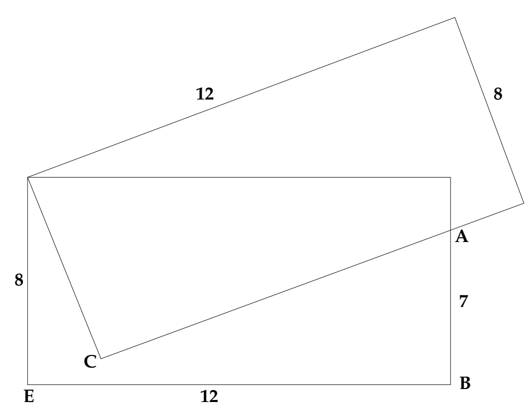
23. Which of the following *could* have a domain which is not a subset of the intersection of the domains of f and g?

(1) $f + g$ (2) $f - g$
(3) $\star f \circ g$ (4) $f \cdot g$
(5) None of the above

24. If $\cos x = .1$, what is $\cos 3x$?

(1) ⋆−.296 (2) −.292
(3) −.294 (4) −.298
(5) None of the above

25. Let two 8×12 rectangles share a common corner and overlap as in the diagram below, so that the distance AB from the bottom right corner of one rectangle to the intersection point A along the right edge of that rectangle is 7. What is the area of the region common to the two rectangles?



(1) 40 (2) ⋆42
(3) 44 (4) 46
(5) None of the above

26. Using the figure from the previous problem, Let the point C be the corner of the slanted rectangle shown. What is the sum of the coordinates of C, given that the lower left corner E of the unslanted rectangle is at $(0, 0)$.

(1) 6.1 (2) 6.2
(3) 6.3 (4) ⋆6.4
(5) None of the above

27. Let A denote the set of all (positive) divisors of 84^4. The product of all the numbers in A equals 84^z for some integer z. What is the value of z?

(1) 430 (2) 440

(3) $\star$450 (4) 460

(5) None of the above

28. Given that $\log_6 2 = .387$, find $\log_6 9$.

(1) 1.116 (2) $\star$1.226

(3) 1.336 (4) 1.446

(5) None of the above

29. How many real numbers x satisfy both of these equations?

$$x^4 + x^3 - x^2 - 2x - 2 = 0$$

and

$$x^4 - x^3 - x^2 + 2x - 2 = 0$$

(1) 0 (2) 1

(3) $\star$2 (4) 3

(5) None of the above

30. Three girls are racing around a track, each at a constant speed. Hannah is the fastest and passes Erin every 10 minutes. On the other hand, Erin passes Jeannie every 15 minutes. How often does Hannah pass Jeannie?

(1) Every 5 minutes (2) $\star$Every 6 minutes

(3) Every 7 minutes (4) Every 8 minutes

(5) None of the above

31. Given that z satisfies

$$z + \frac{1}{z} = 2\cos 13^\circ,$$

how many degrees are in the angle B for which

$$z^2 + \frac{1}{z^2} = 2\cos B?$$

(1) 27 (2) 28

(3) 29 (4) 30

(5) None of the above

32. Evaluate

$$\sin 7^\circ \cos 53^\circ + \sin 8^\circ \cos 38^\circ.$$

(1) $\frac{\sqrt{3}+1}{2}$ (2) $\frac{\sqrt{3}+1}{4}$

(3) $\star\frac{\sqrt{3}-1}{4}$ (4) $\frac{\sqrt{3}-1}{2}$

(5) None of the above

Bonus Questions: Show all your work – use the colored sheets provided by the proctors

1. An equilateral triangle is inscribed in a circle. Let D and E be midpoints of two of its sides, and let F be the point where the line from D through E meets the circle. What is the ratio DE/EF?

2. Find all of the pairs (n, m) satisfying $10 \le m < n$ and $m + n \le 99$ that have the property that $m + n$ and $n - m$ have the same digits in reverse order. You are allowed to count cases such as $(33, 27)$ where the sum and difference are 60 and 06.