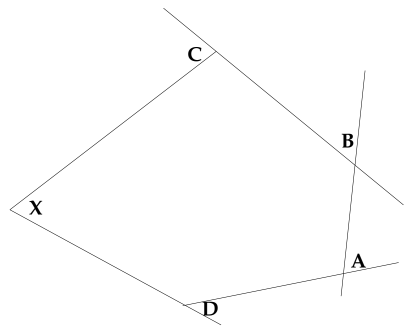
1. Find the sum of the measures of the angles A, B, C, D, X, given that X measures $80°$.



(1) 230 (2) 240

(3) 250 (4) ⋆260

(5) None of the above

2. What is the degree of the polynomial of largest degree which is a factor of both $x^{104}-1$ and $x^{182}-1$?

(1) 20 (2) 22

(3) 24 (4) ⋆26

(5) None of the above

3. If triangle ABC is isosceles, and angle A has measure 100 degrees, what is the degree measure of angle C?

(1) Can't be determined (2) ⋆40

(3) 35 (4) 30

(5) None of the above

4. Find the area of the planar region defined by

$$1 \leq |x| + |y| \text{ and } x^2 - 2x + 1 \leq 1 - y^2.$$

(1) $\star(3/4)\pi$ (2) $\pi/2$

(3) π (4) $\pi/4$

(5) None of the above

5. What fraction is represented in base 5 by the repeating $0.3333\ldots$?

(1) ⋆3/4 (2) 1/2

(3) 2/3 (4) 3/2

(5) None of the above

6. An illegible receipt shows 72 canned hams were purchased for $x67.9y$. What is $x+y$?

(1) 2 (2) ⋆5

(3) 6 (4) 9

(5) None of the above

7. You must form a committee, and you determine that:

1. If Aaron is on the committee, then Barbara wants to be on it.
2. Darron won't serve if Charles is on the committee.
3. You feel that Erin should be on the committee.
4. Aaron and Charles agree that at least one of them should serve.
5. Frank wants to be included only if Barbara and Charles take part.
6. Darron wants to be included if and only if Erin is.

Put these people in alphabetic order and assign them their cardinal numbers (so that Aaron is assigned one and Barbara is assigned two, etc.) What is the sum of the numbers assigned to the people who end up on the committee, if everyone is satisfied?

(1) 9 (2) 10

(3) 11 (4) ⋆12

(5) None of the above

8. Four old cannonballs, each touching the other three, are placed pyramid–like next to a cannon down by the VFW. If each cannonball has radius 10 cm, how high is the top of the topmost cannonball above the ground?

(1) $\star 20 + 20\sqrt{2/3}$ (2) $20 + 20\sqrt{3}$

(3) $20 + 10\sqrt{3}$ (4) $20 + 10\sqrt{2}$

(5) None of the above

9. What is the sum of the digits of the smallest positive integer n such that $n/2$ is a perfect square and $n/3$ is a perfect cube?

(1) $\star 18$ (2) 21

(3) 24 (4) 27

(5) None of the above

10. An algorithm generates a sequence a_1, a_2,, a_3, ... of approximations to the quantity $\sqrt{7}$. The absolute errors satisfy

$$|a_n - \sqrt{7}| < \frac{1}{(n+3)^2}.$$

Based on this information, how large should n be to guarantee an error less than .000001?

(1) 994 (2) $\star 997$

(3) 1003 (4) 1006

(5) None of the above

11. The sum of all products which comprise the multiplication table of the numbers 1 through 2 is 9. What is the sum of all products which comprise a standard multiplication table of the numbers 1 through 20?

(1) $\star$44,100 (2) 44,200

(3) 44,300 (4) 44,400

(5) None of the above

12. What answer do you get if you solve a Wylie Math Tournament problem and notice that your answer is the positive square root of the sum obtained by adding 2 to the answer to problem 12, which reads "What answer do you get if you solve a Wylie Math Tournament problem and notice that your answer is the positive square root of the sum obtained by adding 2 to the answer to problem 12, which reads " ...

(1) 1 (2) $\star 2$

(3) 3 (4) 4

(5) None of the above

13. How many integers from 1 to 1001 are divisible by either 2, 3, or 5?

(1) 701 (2) $\star 734$

(3) 801 (4) 834

(5) None of the above

14. By counting very quickly you notice that your car tire makes 672 revolutions in one mile. What is the radius of the tire rounded to the nearest inch?

(1) 14 (2) $\star 15$

(3) 16 (4) 17

(5) None of the above

15. A farmer spends $4,000 to obtain 100 head of livestock. Prices are: calves – $120 each; lambs – $50 each; piglets – $25 each. If he purchased at least one animal of each type and fewer than 10 calves, how many lambs did he buy?

(1) 44 (2) 46

(3) 48 (4) 50

(5) None of the above

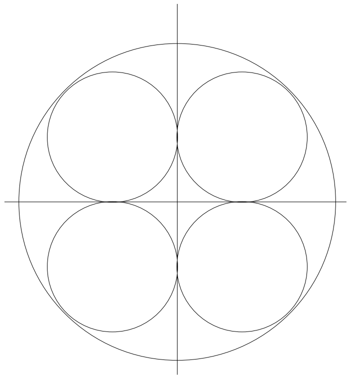
16. How many 3 digit numbers have the property that the 100's digit times the ten's digit is equal to the one's digit?

(1) 28 (2) 29

(3) 31 (4) $\star 32$

(5) None of the above

17. A circular dart board of radius 1 has circular bullseyes of equal size inscribed as shown. If a randomly thrown dart hits the board, what is the probability that the dart misses the bullseyes?



(1) $3\pi/10$ (2) $\pi/4$

(3) $4\sqrt{2}-5$ (4) $\star 8\sqrt{2}-11$

(5) None of the above

18. While waiting for a delayed flight at the Charles DeGaulle airport in Paris, I noticed a man going up a very long escalator. He took 40 seconds to get from the bottom to the top, but he also sped up his process by taking 40 steps along the way. Then, an older gentleman proceeded to go up the same escalator in 50 seconds, taking 20 steps along the way. I then was able to compute how many steps were on the escalator. How many were there?

(1) $\star$120 (2) 130

(3) 140 (4) 160

(5) None of the above

19. The nine numbers 6561, 2187, 729, 243, 81, 27, 9, 3, and 1 are put into a 3×3 grid so that each number appears exactly once in the grid. They are positioned in such a way that the product of any row or any column is the same. What must this product be?

(1) 534141 (2) $\star$531441

(3) 531423 (4) 421423

(5) None of the above

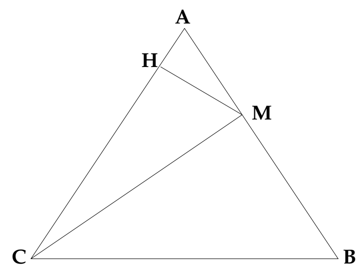
20. If the real number 99π is written as a decimal approximation, what is the one's digit?

(1) 0 (2) $\star$1

(3) 2 (4) 3

(5) None of the above

21. In the following equilateral triangle ABC, CM is an altitude and MH is an altitude of triangle ACM. What is $\frac{HC}{HA}$?



(1) 1/2 (2) $\star$1/3

(3) 1/4 (4) 1/5

(5) None of the above

22. A certain state's license plates have the form $\diamondsuit\diamondsuit\ \spadesuit\spadesuit\ \diamondsuit\diamondsuit$, where $\diamondsuit$ represents a numerical digit and $\spadesuit$ represents a letter. How many palindromic license plates does this state have?

(1) 2600 (2) 2600^2

(3) 50,000 (4) More than 7,000,000

(5) None of the above

23. Which of the following are equivalent to

$$-\sqrt{5-2\sqrt{6}}+\sqrt{5+2\sqrt{6}}?$$

(1) $\star 2\sqrt{2}$ (2) $\sqrt{10}$

(3) $2\sqrt{3}$ (4) 4

(5) None of the above

24. In a 10 team baseball league, each team plays each of the others 18 times. The season ends, not in a tie, with each team the same number of games ahead of the following team. The greatest number of games the last place team could have won is:

(1) 70 (2) $\star$72

(3) 74 (4) 76

(5) None of the above

25. I have an insurance policy that pays for 80% of all expenses over the first \$100 of expenses. If my insurance will pay for all but \$176, the the total of my expenses is:

(1) \$460 (2) \$470

(3) ⋆\$480 (4) \$490

(5) None of the above

26. If $\dfrac{x+2y}{2x+y} = 3$, what is the value of $\dfrac{x+4y}{4x+y}$?

(1) 18 (2) ⋆19

(3) 20 (4) 21

(5) None of the above

27. How many numbers are common to the first 100 terms of the arithmetic progression

$$1, 4, 7, 10, 13, 16, 19, \ldots$$

and to the first 100 terms of the arithmetic prgression

$$2, 6, 10, 14, 18, 22, 26, \ldots?$$

(1) 22 (2) 23

(3) 24 (4) ⋆25

(5) None of the above

28. Two sides of a parallelogram are 3 and 5, and one diagonal is 4. Find the area of the parallelogram.

(1) 10 (2) ⋆12

(3) 14 (4) 16

(5) None of the above

29. If all the integers from 1 to 1,000 were written out, how many times would the digit 9 appear?

(1) 240 (2) 260

(3) 280 (4) ⋆300

(5) None of the above

30. How many real numbers x satisfy both of these equations?

$$x^4 + x^3 - x^2 - 2x - 2 = 0$$

and

$$x^4 - x^3 - x^2 + 2x - 2 = 0$$

(1) 0 (2) 1

(3) ⋆2 (4) 3

(5) None of the above

31. Three girls are racing around a track, each at a constant speed. Hannah is the fastest and passes Erin every 10 minutes. On the other hand, Erin passes Jeannie every 15 minutes. How often does Hannah pass Jeannie?

(1) Every 5 minutes (2) ⋆Every 6 minutes

(3) Every 7 minutes (4) Every 8 minutes

(5) None of the above

32. What is the coefficient of x^2 in the expansion of

$$\left(2x + \frac{1}{2x}\right)^6?$$

(1) ⋆60 (2) 62

(3) 64 (4) 66

(5) None of the above

Bonus Questions: Show all your work – use the colored sheets provided by the proctors.

1. An equilateral triangle is inscribed in a circle. Let D and E be midpoints of two of its sides, and let F be the point where the line from D through E meets the circle. What is the ratio DE/EF?

2. Find all of the pairs (n, m) satisfying $10 \le m < n$ and $m + n \le 99$ that have the property that $m + n$ and $n - m$ have the same digits in reverse order. You are allowed to count cases such as $(33, 27)$ where the sum and difference are 60 and 06.