1. A 14 game winning streak raises the Greenville Drive's winning "percentage" from .500 to .625. How many games had the Drive played prior to the beginning of the streak?

(1) 43 (2) 44

(3) 45 (4) 46

(5) None of the above

2. A line passes through $(2, 2)$ and cuts a triangle of area 9 square units from the first quadrant. What is the sum of all possible values for the slope of such a line?

(1) -2.5 (2) -2

(3) -1.5 (4) -1

(5) None of the above

3. If you have ten dimes, twenty nickels, and one hundred pennies, in how many ways can you pay out one dollar?

(1) 100 (2) 111

(3) 121 (4) 132

(5) None of the above

4. What is the radius of the smallest circle that contains both of the circles $x^2 + y^2 = 4$ and $(x-3)^2 + (y-3)^2 = 9$?

(1) $\frac{3\sqrt{3}+5}{2}$ (2) $\frac{3\sqrt{3}+7}{2}$

(3) $\frac{3\sqrt{2}+5}{2}$ (4) $\frac{3\sqrt{2}+7}{2}$

(5) None of the above

5. Find the sum of the reciprocals of all integers of the form $2^a 3^b$ where a and b are nonnegative integers.

(1) 3 (2) π

(3) $8\sqrt{3}/3$ (4) 3.5

(5) None of the above

6. How many mutually non-threatening bishops can you put on a standard 8×8 chessboard? (A bishop can move to any square along any diagonal from its current position.)

(1) 8 (2) 10

(3) 12 (4) 14

(5) None of the above

7. When Perri is as old as Malorie is now, Malorie will be 6 years older than Perri was when Malorie was as old as Perri is now. When Perri is twice as old as Malorie is now, Malorie will be twice as old as Perri will be when Malorie is twice as old as Perri is now. What is the sum of Perri and Malorie's ages now?

(1) 12 (2) 14

(3) 6 (4) 18

(5) None of the above

8. Simplify

$$2 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{\ddots}}}}.$$

(1) $1 + \frac{2\sqrt{3}}{3}$ (2) $1 + \frac{\sqrt{13}}{3}$

(3) $1 + \frac{\sqrt{14}}{3}$ (4) $1 + \frac{\sqrt{15}}{3}$

(5) None of the above

9. What is the remainder when the base 9 number

122333444455555666666777777788888888

is divided by 8?

(1) 0 (2) 2

(3) 4 (4) 6

(5) None of the above

10. A marble of diameter 2 is dropped into a conical cup of height 4 and radius 3. When it gets stuck, how high is the bottom of the marble above the vertex of the cone?

(1) 1/3 (2) $\sqrt{2}/3$

(3) 2/3 (4) $\sqrt{3}/3$

(5) None of the above

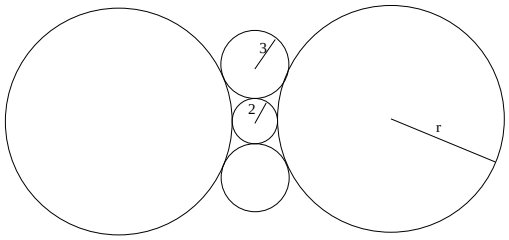
11. If Nancy gave Ronald a third of her jelly beans, Ronald would have twice as many jelly beans as Nancy would have if Ronald gave Nancy a third of his jelly beans. If Nancy gave Ronald a third of her jelly beans and then Ronald ate eight jelly beans, Ronald would have three times as many jelly beans as Nancy would have if Ronald gave Nancy a third of his jelly beans and then Nancy ate eight jelly beans. How many jelly beans does Ronald have?

(1) 24 (2) 26

(3) 28 (4) 30

(5) None of the above

12. A circle of radius 2 is surrounded and tangent to four mutually tangent circles with radii 3, r, 3, and r (given counterclockwise). Find r.



(1) $7\sqrt{2}$ (2) $6\sqrt{3}$

(3) 9 (4) 10

(5) None of the above

13. What is the least number of people that can be at a gathering so that the probability that two have birthdays in the same month exceeds 1/2? (Assume that birthdays occur with equal probability in each month.)

(1) 5 (2) 6

(3) 7 (4) 8

(5) None of the above

14. In base b, c^2 is written 10. How do you write b^2 in base c?

(1) 100 (2) 1000

(3) 10000 (4) 100000

(5) None of the above

15. Abigail, Beth, Chinue, and Damisi are playing rounds of a certain game. Whenever a player wins a round, she pays each of the other players exactly the amount that they have in their posession. That is, the winner doubles the money of each loser. After four rounds, it turned out that each had won exactly one round, and the winning took place in alphabetic order of their names. Furthermore, at the end of these four rounds, each player had exactly \$16. How much did Abigail have at the beginning of the game?

(1) \$35 (2) \$37

(3) \$39 (4) \$41

(5) None of the above

16. A roll of toilet paper has a diameter of 6 inches. The tissue is rolled around a 2 inch diameter carboard tube. There are 1000 sheets, each 4 inches by 4 inches. Estimate, to the nearest thousandth of an inch, the thickness of each sheet of paper.

(1) .003 (2) .004

(3) .005 (4) .006

(5) None of the above

17. A solved Rubik's cube is a cube whose faces are each painted a different color. Six fixed colors are used in constructing a Rubik's cube, though you may have noticed that they are not always painted on in the same order. How many different ways can the cube be colored? (Two 'solved' cubes are colored the same if a rotation of one whole cube makes it look exactly like the other.)

(1) 15 (2) 30

(3) 120 (4) 360

(5) None of the above

18. If the numbers a and b are chosen without replacement from the set $\{2, 4, 6, 8, 10\}$, what ordered pair (a, b) would give the largest value to

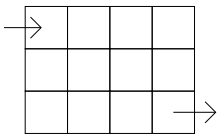
$$\log_a(\log_b(25))?$$

(1) $(4, 2)$ (2) $(2, 4)$

(3) $(2, 10)$ (4) $(10, 2)$

(5) None of the above

19. How many paths are there through a 3×4 grid given that each of the 12 squares must be visited exactly once and the initial entrance and final exit are as shown? Only horizontal or vertical moves are allowed.



(1) 4 (2) 5

(3) 6 (4) 7

(5) None of the above

20. Ms. Lyddane gave the same algebra test to her three classes. The first class averaged 80%, the second class averaged 85%, and the third 89%. Together, the grades from the first two classes averaged 82%, while the grades from the second and third together averaged 87%. What is the average for all three classes combined?

(1) 84% (2) 85%

(3) 86% (4) 87%

(5) None of the above

21. How many times in a 24-hour day are the minute and the hour hands of a clock perpendicular?

(1) 24 (2) 36

(3) 44 (4) 48

(5) None of the above

22. Evaluate

$$\frac{(2+1)(2^2+1)(2^4+1)(2^8+1)(2^{16}+1)+1}{2^{33}}.$$

(1) 1/4 (2) 1/2

(3) 1 (4) 2

(5) None of the above

23. I have 3 dice: one red, one blue, and one green, and I roll all three simultaneously. Let p be the probability that the sum of the numbers on the red and the blue dice is equal to the number on the green die. If p is written in lowest terms as a/b, what is $a+b$?

(1) 79 (2) 81

(3) 83 (4) 85

(5) None of the above

24.

$$\left(\sqrt[3]{3} + i \cdot 3^{5/6}\right)^3$$

is an integer. What is it?

(1) -21 (2) -22

(3) -23 (4) -24

(5) None of the above

25. Beginning with a tank containing 8075 gallons of gasoline, more gas is added at a rate of 5 gallons per minute, while alcohol is added at a rate of 10 gallons per minute. When the mixture is 10% alcohol, how many gallons of the mixture are in the tank?

(1) 9350 (2) 9400

(3) 9450 (4) 9500

(5) None of the above

26. What is the largest power of 42 which divides 2006! evenly?

(1) 331 (2) 332

(3) 333 (4) 334

(5) None of the above

27. I have a large round cake which I want to cut into as many pieces as possible. However, I refuse to do anything but a straight chord-like cut, and I refuse to make more than 10 cuts. What is the largest number of pieces of cake I can make?

(1) 55 (2) 56

(3) 57 (4) 58

(5) None of the above

28. Let $A = 1^{-4} + 2^{-4} + 3^{-4} + 4^{-4} + \cdots$ and $B = 1^{-4} + 3^{-4} + 5^{-4} + 7^{-4} + \cdots$. What is the ratio A/B?

(1) 16/15 (2) 17/16

(3) 18/17 (4) 19/18

(5) None of the above

29. What is the remainder when $4^{87} + 6^{87}$ is divided by 25?

(1) 19 (2) 20

(3) 21 (4) 22

(5) None of the above

30. What is the number of degrees in the acute angle θ satisfying $\cos(\theta) = \frac{1}{2}\sqrt{2+\sqrt{2}}$?

(1) 20 (2) 22

(3) 24 (4) 26

(5) None of the above

31. In my living room at home I have a model of the first quadrant laid out on the floor. My wife and I like to play the following game with a "yardstick" cut to be exactly 10 feet long. She picks a real number b between 0 and 10 along the y axis, and lays the yardstick down with the tip at the point $(0, b)$ so as to form a right triangle with the yardstick forming the hypotenuse, and the two axes forming the other two sides. She wins the game if the area of the triangle formed is greater than 15. If she picks her number at random from the interval $[0, 10]$ (with uniform distribution), what is the probability that she will win?

(1) 3/5 (2) $\sqrt{10}/5$

(3) $\sqrt{11}/5$ (4) $2\sqrt{3}/5$

(5) None of the above

32. Let n be the smallest postive integer larger than 150 so that $\binom{n}{151}$ is divisible by $\binom{n}{150}$, but is not equal to it. What is the sum of the digits of n?

(1) 8 (2) 9

(3) 10 (4) 11

(5) None of the above

Bonus Questions: Show all your work.

1. What is the value of the positive integer n for which the least common multiple of 36 and n is 500 greater than the greatest commmon divisor of 36 and n?

2. Three vertices of a cube in space have coordinates $(3, 4, 1)$, $(5, 2, 9)$, and $(1, 6, 5)$. What are the coordinates of the center of the cube?