Furman University
Wylie Mathematics Tournament
Ciphering Competition

March 11, 2006

House Rules

1. All answers are integers(!)
2. All answers must be written in standard form. For example, 8 not 2^3, and 10, not $\binom{5}{2}$.

Division II Round I Ciphering

Participants in Round I ciphering from Division II schools should now make their way to the front.

Division II Round I – Number 1

An isosceles triangle has two sides of length 10 and one side of length 12. What is its area?

Division II Round I – Number 2

Suppose that $x = .12\overline{30}$ is written in the form a/b where a and b are positive integers and this fraction is in reduced form. What is $a + b$?

Division II Round I – Number 3

Three positive integers add to 18. The first is twice the second, and the third is six more than the second. What is the largest of these numbers?

Division II Round I – Number 4

What number is one-half of one-quarter of one-tenth of one-half of 2400?

Division II Round I – Number 5

For what digit d is the number $16d1$ a perfect square?

Division II Round I – Number 6

In what base b does

$$23 \cdot 14 = 333?$$

Division II Round I – Number 7

What is the coefficient of x^7 in the polynomial expansion of this expression:

$$(1 + 2x + x^2)^4?$$

Division II Round II Ciphering

Participants in Round II ciphering from Division II schools should now make their way to the front.

Division II Round II – Number 1

Suppose that $f(x) = ax^7 + bx^3 + cx - 5$, and that $f(-7) = 9$. What is $f(7)$?

Division II Round II – Number 2

If the ratio of $y + 5x$ to $x + 2y$ is $2/3$, what is the ratio of y to x?

Division II Round II – Number 3

What is the remainder when $x^{51} + 51$ is divided by $x + 1$?

Division II Round II – Number 4

While working at Mal-Wart, Ben marked an item up by 30%, but then Amy came along and marked it up by another 30%. What is now the percentage total markup over the original price?

Division II Round II – Number 5

If $f(x) = 1 - \frac{1}{1-x}$, what is the value of the following expression:

$$f(f(\cdots(f(3))\cdots))$$

where there are 2006 $f's$ in the composition?

Division II Round II – Number 6

How many positive integers less than the base 5 number 432 are divisible by 4?

Division II Round II – Number 7

A certain geometric progression has its first and third terms sum to 40, and its second and fourth terms sum to 80. What is the first term in the progression?

Division II Round III Ciphering

Participants in Round III ciphering from Division II schools should now make their way to the front.

Division II Round III – Number 1

Last night I made mashed potatoes. When it was time to pass them, I first passed the bowl to my daughter Darby, who took 70% of what was available. She then passed the bowl to my other daughter Hannah who also took 70% of what was available. When the potatoes got to me, I noticed that there were only 2.7 ounces left. How many ounces of potatoes were in the bowl originally?

Division II Round III – Number 2

The sum of the digits of a 3-digit number is 15. The unit's digits is one more than twice the hundred's digit, and the ten's digit is the average of the other two digits. What is the number?

Division II Round III – Number 3

What is the unit's digit of the number 7^{2006}?

Division II Round III – Number 4

If x and y are both positive integers less than 50, what is x if $x^2 - y^2 = 125$?

Division II Round III – Number 5

Mark rides up a hill on his bicycle at 9 mph, and rides down the same hill at 21 mph. If his average speed for the trip is x mph, what is $\lfloor x \rfloor$?

Division II Round III – Number 6

I have 7 white and 4 black socks in my drawer. I have to pick my socks out in the dark, because I get up really early, and I don't want to disturb my sleeping spouse. If I pick two socks at random, Let x be the probability that the socks match color. If x is written in the form a/b in reduced form with a and b positive integers, what is $a + b$?

Division II Round III – Number 7

In what base b does this expression hold:

$$4 \cdot 12 = 52.$$

Division I Round I Ciphering

Participants in Round I ciphering from Division I schools should now make their way to the front.

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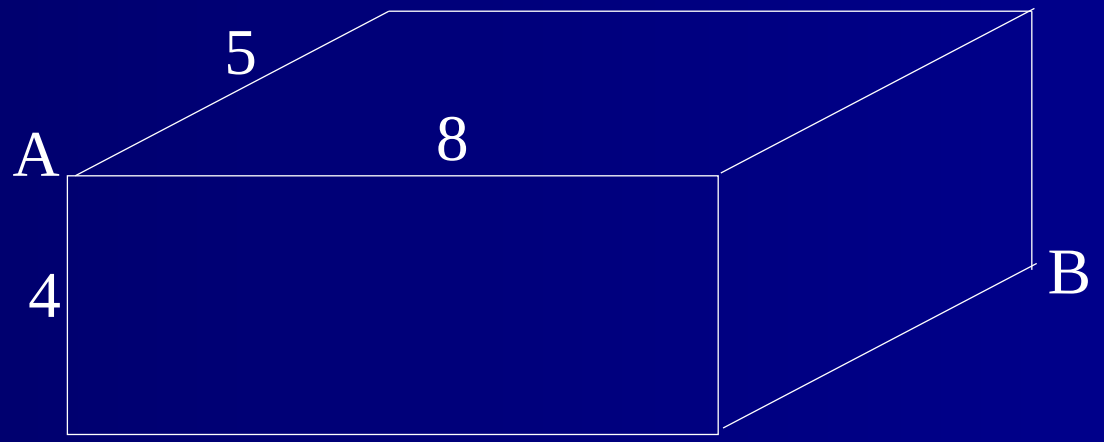
Division I Round I – Number 1

How many positive integers w have $\log_w(256) = n$ where n is a positive integer?

Division I Round I – Number 2

A spider is going to walk from point A to point B on the surface of the pictured box, using the quickest path possible. He walks at one unit per second. How long will it take him to get from A to B, in seconds?



Division I Round I – Number 3

Point W is outside of square $RSTU$ so that triangle RSW is equilateral. What is the measure of angle TWU in degrees?

Division I Round I – Number 4

Find this product:

$$\log_5(169) \cdot \log_{13}(625).$$

Division I Round I – Number 5

The largest of seven consecutive *even* integers is twice the smallest. What is the smallest?

Division I Round I – Number 6

On my farm I have horses, chickens, 5 cats and a dog. Altogether, my animals have 100 heads and 250 legs. How many horses do I have?

Division I Round I – Number 7

Find the sum of all integers k so that $1 \leq k \leq 100$, and k is not divisible by 4.

Division I Round II Ciphering

Participants in Round II ciphering from Division I schools should now make their way to the front.

Division I Round II – Number 1

For assemblies at my high school, 600 students are usually seated in rows with an equal number of students per row. If 5 chairs were added to each row, 6 fewer rows would be needed. How many students usually sit in each row?

Division I Round II – Number 2

Given this expression:

$$a_n = a_{n-1} + a_{n-2} + a_{n-3}, \text{ for } n \geq 4,$$

with $a_3 = -25$, $a_4 = 10$, and $a_7 = 37$. Find a_6.

Division I Round II – Number 3

Mickey beats Bob in a single game of backgammon with probability 2/3. A tournament consists of “the best two out of three.” The probability that Mickey wins a tournament against Bob is written in reduced form as a/b with a and b positive integers What is $a + b$?

Division I Round II – Number 4

Evaluate this expression:

$$\left(\sqrt{5-2\sqrt{6}}-\sqrt{5+2\sqrt{6}}\right)^2.$$

Division I Round II – Number 5

In the game of Killer Ball, one can either score a BINK which is worth 3 points or a BLONK which is worth 7 points. How many positive integers n exist for which it is true that a score of n points can't be achieved in a game of Killer Ball?

Division I Round II – Number 6

Suppose that W, M, and D are distinct positive integers so that $W \cdot M \cdot D = 2006$. What is the largest possible value of $W + M + D$?

Division I Round II – Number 7

You may have noticed while solving the last problem that 2006 is the product of exactly three primes: $2006 = 2 \cdot 17 \cdot 59$. There is exactly one other year in this decade which is the product of three distinct primes. What is the sum of the three primes which are factors of that year? Hint: The year in question contained no Olympics.

Division I Round III Ciphering

Participants in Round III ciphering from Division I schools should now make their way to the front.

Division I Round III – Number 1

My car insurance pays 80% of the cost of repairs, after a $100 deductible is subtracted. I recently had some work done, and after my insurance company paid their part, the repair company billed me $176 — which was the outstanding balance. What was the dollar amount of the bill before my insurance company paid their part?

Division I Round III – Number 2

If six teenagers can slurp down 12 ice cream cones in 30 minutes, how many minutes will it take 10 teenagers to finish off 40 cones?

Division I Round III – Number 3

After a trip to Haywood Mall, Hannah had change from her $20 bill. She noticed that she had the same number of pennies, nickels and quarters, and that these totaled $3.10. How many of each coin does she have?

Division I Round III – Number 4

State the largest of these three numbers in the form required by the house rules:

$$7^7,\ 6^8,\ 4^{10}$$

Division I Round III – Number 5

Darby is now half as old as Russell was six years ago. Three years ago, Russell was three times as old as Darby. In six years, Russell will be twice as old as Darby will be then. How old is Darby now?

Division I Round III – Number 6

Consider this sequence:

$$\frac{2}{1}, \frac{5}{4}, \frac{10}{9}, \frac{17}{16}, \ldots \frac{n^2+1}{n^2}, \ldots.$$

What is the smallest value of n so that the nth term of this sequence is less than 1.001?

Division I Round III – Number 7

What is the ten's digit of 2^{100}?

That’s All, Folks

Awards Ceremony to follow soon. Please be patient while we tally the results.