1. How many different 10–letter arrangements of the letters in the "word" GOPALADINS are there?

(1) $10!$ (2) $\dfrac{10!}{8!2!}$

(3) $\dfrac{10!}{2!}$ (4) $8!$

(5) None of the above

2. What is the solution set of

$$|2x-5| > |2x+3|?$$

(1) ϕ (2) $\{x : x \leq -3/2\}$

(3) $\{x : x < 1/2\}$ (4) $\{x : -3/2 \leq x < 1/2\}$

(5) None of the above

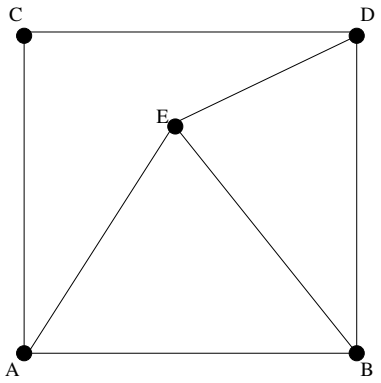
3. For how many positive integers n is there a triangle with three positive acute angles and sides of lengths 10, 24, and n?

(1) 3 (2) 4

(3) 5 (4) 6

(5) None of the above

4. $ABCD$ is a square, and ABE is an equilateral triangle. The measure of angle CDE in radians is:



(1) $\frac{\pi}{15}$ (2) $\frac{\pi}{12}$

(3) $\frac{\pi}{10}$ (4) $\frac{\pi}{9}$

(5) None of the above

5. What is the sum of the squares of the three solutions to the equation

$$x^3 + x^2 + x + 1 = 0?$$

(1) -1 (2) 0

(3) 1 (4) 2

(5) None of the above

6. Suppose that $\star$ is an operation on positive real numbers defined by $a \star b = a^{b-1}$. What is $3 \star (2 \star 3)$?

(1) 27 (2) 9

(3) 3 (4) 1

(5) None of the above

7. Which of the following can be said about the equation

$$|x^2| + |x| - 6 = 0?$$

(1) There is only one root

(2) There are four roots

(3) The product of the roots is -4

(4) The product of the roots is -6

(5) None of the above

8. What type of conic section is the graph of

$$2x^2 + 3y^2 - 16x - 12y = -44?$$

(1) hyperbola (2) ellipse

(3) circle (4) directrix

(5) None of the above

9. If $\sin(x) = .4$ and $\pi/2 < x < \pi$, then what is the value of $\cos(2x)$?

(1) .24 (2) .42

(3) .48 (4) .68

(5) None of the above

10. The sum of a certain number of positive integers is 31. What is the biggest their product can be?

(1) 55296 (2) 78732

(3) 118098 (4) 49152

(5) None of the above

11. I have three weights: a one–pound weight, a five–pound weight, and a fourteen-pound weight. I also have a two-pan balance scale. I can put weights together with what I am trying to weigh in either pan of the scale. Let U be the set of all prime numbers x less than 20 with the property that x is a weight which can't be determined using these tools. How many elements are there in U?

(1) 4 (2) 5

(3) 6 (4) 7

(5) None of the above

12. With 150 members of the Furman faculty voting, a proposal to change to a semester system was defeated. After a slight amendment, a revote resulted in the passage of the proposal by 2/3 of the margin by which it was originally defeated. The number voting for the proposal on the revote was 17/18 of the number voting against it orginally. How many more faculty members voted for the proposal the second time than voted for it the first time?

(1) 10 (2) 15

(3) 20 (4) 25

(5) None of the above

13. Suppose that $w = (0.001)^{1000}$, $x = (0.001)^{0.001}$, $y = (1.001)^{1000}$, and $z = (2^{1000} - 1)^{0.001}$. Put these numbers in order from smallest to largest.

(1) w, x, y, z (2) w, x, z, y

(3) x, w, y, z (4) x, w, z, y

(5) None of the above

14. Suppose that

$$3 = \log_a(\log_2(a)) \cdot \log_2(a).$$

What is a?

(1) 128 (2) 225

(3) 250 (4) 256

(5) None of the above

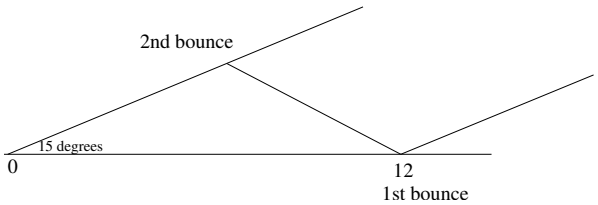
15. What is the ratio of the volume of a cube circumscribed about a sphere to the volume of a cube inscribed inside the same sphere?

(1) $\sqrt{2}$ (2) $\sqrt{3}$

(3) $2\sqrt{2}$ (4) $3\sqrt{3}$

(5) None of the above

16. Two walls meet at a fifteen degree angle, as shown. An incoming particle enters parallel to one wall and bounces off the other wall 12 feet from the vertex. Assuming perfect bounces, how many feet from the vertex will the particle be on the 11th bounce?



(1) 1 (2) 2

(3) 4 (4) 8

(5) None of the above

17. A certain right triangle has its area numerically equal to its perimeter. The length of each side is an even integer. What is the perimeter?

(1) 26 (2) 28

(3) 30 (4) 32

(5) None of the above

18. It is the bottom of the ninth and the Furman baseball team must score a run to avoid losing the game. There are no outs and a runner is on first base. Records show that a run is scored in such circumstances 40% of the time. Perhaps the runner should try to steal second base, for with a runner on second and no outs a run is scored 60% of the time. However, if the stealing attempt fails, there will be one out and nobody on base, in which case the chances of scoring a run is reduced to 15%. If the runner is successful in stealing with probability p, how large must p be in order for an attempted steal to be a good strategy?

(1) 4/9 (2) 5/9

(3) 2/3 (4) 7/9

(5) None of the above

19. How many of the coefficients in the expansion of $(x+y)^{64}$ are even numbers?

(1) 33 (2) 49

(3) 61 (4) 63

(5) None of the above

20. What is

$$(\cos\pi/8 + i\sin(\pi/8))^4(\cos\pi/15 + i\sin\pi/15)^5?$$

(1) $\sqrt{3}/2 + i/2$ (2) $\sqrt{3}/2 - i/2$

(3) $-\sqrt{3}/2 + i/2$ (4) $-\sqrt{3}/2 - i/2$

(5) None of the above

21. Assume that $a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$ is the remainder when $x^{19}+2x^{14}+3x^9+4x^4+5$ is divided by $x^5 - x^4 + x^3 - x^2 + x - 1$. What is $\sum_{i=0}^{4} a_i$?

(1) 15 (2) 16

(3) 17 (4) 18

(5) None of the above

22. If $(333, 333, 333, 333)^2$ is written as a standard integer in base 10, what is the sum of the digits of this number?

(1) 108 (2) 117

(3) 124 (4) 126

(5) None of the above

23. Suppose that $w = 2^{1/2}$, $x = 3^{1/3}$, $y = 6^{1/6}$, and $z = 8^{1/8}$. From among these numbers, list the biggest first and the second biggest second.

(1) w, x (2) x, w

(3) y, z (4) x, z

(5) None of the above

24. A and B are points on a circle with center C. Angle ACB has measure 30 degrees. If a point D is chosen at random on the circle, what is the probability that triangle ABD is obtuse?

(1) 2/3 (2) 3/4

(3) 5/6 (4) 11/12

(5) None of the above

25. The set of points (x, y) whose distance from the line $y = 2x + 2$ is the same as the distance from (x, y) to $(2, 0)$ is a parabola. This parabola is congruent to a parabola in standard form $y = kx^2$ for some k. What is k?

(1) $\sqrt{5}/12$ (2) $\sqrt{6}/12$

(3) $\sqrt{7}/12$ (4) $\sqrt{8}/12$

(5) None of the above

26. How many zeroes are at the end of the number 2005!?

(1) 498 (2) 499

(3) 500 (4) 501

(5) None of the above

27. Suppose that a, b, c, d are distinct digits chosen from $\{2, 3, 4, 5, 8, 9\}$ so that the following multiplication is valid:

$$\begin{array}{cccccc} & & & 7 & a & d \\ \times & & & & b & a \\ \hline & & b & b & 0 & b \\ & 1 & d & 6 & c & 0 \\ \hline 1 & 6 & c & c & b & \end{array}$$

What is $a^2 - bd$?

(1) 1 (2) 3

(3) 5 (4) 7

(5) None of the above

28. A quartic polynomial p has roots -2, 3, 5, and 7. If $p(0) = 1$, what is the remainder when $p(x)$ is divided by $x - 1$?

(1) 18/23 (2) 14/29

(3) 6/7 (4) 24/35

(5) None of the above

29. What is the sum of all positive integers z for which there exists a positive integer w with $z^2 - w^2 = 2005$?

(1) 1200 (2) 1206

(3) 1212 (4) 1218

(5) None of the above

30. Note that $100 - 11(3)^2 = 1$. find an the smallest integer b with $100 < b < 1,000$ so that $b^2 - 11a^2 = 1$ for some positive integer $a < b$.

What is $a + b$?

(1) 259 (2) 342

(3) 457 (4) 501

(5) None of the above

31. If n is the smallest 4-digit number so that n^3 and n end in the same 4 digits in base 10, find the sum of the digits of n.

(1) 13 (2) 14

(3) 15 (4) 16

(5) None of the above

32. If

$$n = (123,456,789)(76,543,211) + (23,456,789)^2,$$

what is the sum of the digits of n?

(1) 2 (2) 5

(3) 14 (4) 19

(5) None of the above

Bonus Questions: Show all your work.

1. Compute the sum $A = \frac{1}{5} + \frac{1}{25} + \frac{2}{125} + \frac{3}{625} + \frac{5}{3125} + \cdots$, where each numerator is the sum of the two preceding numerators, and each denominator is 5 times the preceding one. Show your work.

2. Show the work that you did in answering number 17 on this test.