1. If $\dfrac{2^{2005}+2^{2002}}{2^{2004}-2^{2003}}$ is written in the form a/b where a and b are integers with no common divisors, what is $a+b$?

(1) 11 (2) 12

(3) 13 (4) 14

(5) None of the above

2. If $\dfrac{14x-30}{3x^2-27} = \dfrac{A}{x-3} + \dfrac{B}{x+3}$, then $3A-B$ is:

(1) -2 (2) 1

(3) -1 (4) 2

(5) None of the above

3. If $ab = z$ and $1/a^2 + 1/b^2 = w$, which of the following are equal to $(a+b)^4$?

(1) $(z+2w)^2$ (2) $\frac{1}{z^2} + 2w$

(3) $z^2w^2(z+2)^2$ (4) $z^2(z^2w^2+4+4zw)$

(5) None of the above

4. How many natural numbers n satisfy

$$n^2 < 9n < 50n < n^3?$$

(1) 0 (2) 1

(3) 2 (4) 3

(5) None of the above

5. How many 4-digit base 10 numbers x have the property that $x-98$ is divisible by 100?

(1) 72 (2) 81

(3) 90 (4) 99

(5) None of the above

6. If the repeating decimal $.\overline{36}$ is expressed as a rational number a/b with a and b integers in reduced form, then $a+b$ is:

(1) 15 (2) 16

(3) 17 (4) 18

(5) None of the above

7. The product of two certain positive integers is 25 times their quotient. What can you say for sure about this situation?

(1) The sum of the numbers is at least 10

(2) The difference of the numbers is at most 10

(3) One of the numbers is 5

(4) Nothing can be said for sure

(5) None of the above

8. At a local burger joint, you have your choice of real meat or a tofu substitute, and any number of toppings from the following list: { lettuce, tomato, cheese, pickles, onions, ketchup, mustard, mayonnaise}. How many different burgers could one order?

(1) 512 (2) 1024

(3) 40320 (4) 80640

(5) None of the above

9. In November, 2% more new cars were sold than in October, but 2% fewer new cars were sold in December than in November. If 250,000 new cars were sold in October, how many were sold in December?

(1) 250,000 (2) 249,900

(3) 248,000 (4) 247,750

(5) None of the above

10. What is the solution set of

$$|2x-5| > |2x+3|?$$

(1) ϕ (2) $\{x : x \le -3/2\}$

(3) $\{x : x < 1/2\}$ (4) $\{x : -3/2 \le x < 1/2\}$

(5) None of the above

11. Consider the following computer program, written in "pseudocode", whose input is a positive integer n.

1. If n is even, $m = n/2$
2. If n is odd, $m = 3n + 1$
3. Let $n = m$
4. Print n
5. If $n \neq 1$, go to 1
6. Stop

If the number 20 is input into this program, what is the sum of the numbers printed by the program?

(1) 44 (2) 45

(3) 46 (4) 47

(5) None of the above

12. What is the distance between the points of intersection of the graphs of $y - x = -1$ and $y = x^2 - 2x - 1$?

(1) $\sqrt{18}$ (2) $\sqrt{19}$

(3) $\sqrt{20}$ (4) $\sqrt{21}$

(5) None of the above

13. The measure of a certain angle is twice its supplement. What is the degree measure of this angle?

(1) 30 (2) 60

(3) 120 (4) 140

(5) None of the above

14. Suppose that $\star$ is an operation on positive real numbers defined by $a \star b = a^{b-1}$. What is $3 \star (2 \star 3)$?

(1) 27 (2) 9

(3) 3 (4) 1

(5) None of the above

15. How many real solutions does the equation

$$\frac{1}{16} \cdot 2^{x^2} = 2^{3x}$$

have?

(1) 0 (2) 1

(3) 2 (4) 3

(5) None of the above

16. How many real solutions does the equation

$$\frac{2}{x+2} + \frac{1}{x-3} = \frac{5}{x^2 - x - 6}$$

have?

(1) 0 (2) 1

(3) 2 (4) 3

(5) None of the above

17. If $f(n) = \frac{1}{3}n(n+1)(n+2)$, then $f(r) - f(r-1)$ is

(1) $r(r+1)$ (2) $\frac{1}{3}(r+1)(r+2)$

(3) $(r+1)(r+2)$ (4) $\frac{1}{3}r(r+1)$

(5) None of the above

18. The number 102011 in base 3 is equal to the number 455 in base n. What is n?

(1) 5 (2) 7

(3) 9 (4) 11

(5) None of the above

19. Given 50 straight lines in the plane, no three of which have a common point and no two of which are parallel, how many regions is the plane divided into?

(1) 1226 (2) 1275

(3) 1276 (4) 1327

(5) None of the above

20. Goats are attached by a rope to the opposite outside corners of small shed, which is in the middle of a big field. The shed is 3 feet × 6 feet. One goat's rope is 3 feet long while the other's is 6 feet long. What is the total area that the two goats can graze, measured in square feet?

(1) 30π (2) 32π

(3) 34π (4) 36π

(5) None of the above

21. Suppose that

$$f(x)f(y) - f(xy) = x + y$$

for all x and y. Which of the following must be true?

(1) The values of f are always positive

(2) The graph of f is a line of slope 2

(3) The graph of f is *not* a line

(4) $f(x) = x + 1$

(5) None of the above

22. Suppose that $w = (0.001)^{1000}$, $x = (0.001)^{0.001}$, $y = (1.001)^{1000}$, and $z = (2^{1000} - 1)^{0.001}$. Put these numbers in order from smallest to largest.

(1) w, x, y, z (2) w, x, z, y

(3) x, w, y, z (4) x, w, z, y

(5) None of the above

23. Suppose that $w = 2^{1/2}$, $x = 3^{1/3}$, $y = 6^{1/6}$, and $z = 8^{1/8}$. From among these numbers, list the biggest first and the second biggest second.

(1) w, x (2) x, w

(3) y, z (4) x, z

(5) None of the above

24. I have three weights: a one–pound weight, a five–pound weight, and a fourteen-pound weight. I also have a two-pan balance scale. I can put weights together with what I am trying to weigh in either pan of the scale. Let U be the set of all prime numbers x less than 20 with the property that x is a weight which can't be determined using these tools. How many elements are in U?

(1) 4 (2) 5

(3) 6 (4) 7

(5) None of the above

25. The sum of a certain number of positive integers is 31. What is the biggest their product can be?

(1) 55296 (2) 78732

(3) 118098 (4) 49152

(5) None of the above

26. The product of four consecutive integers is $k^2 - 1$, for an integer k. If n is the smallest of the four numbers, what is $n^2 + 3n + 1$, in terms of k?

(1) $k - 2$ (2) $k - 1$

(3) k (4) $k + 1$

(5) None of the above

27. With 150 members of the Furman faculty voting, a proposal to change to a semester system was defeated. After a slight amendment, a revote resulted in the passage of the proposal by 2/3 of the margin by which it was originally defeated. The number voting for the proposal on the revote was 17/18 of the number voting against it orginally. How many more faculty members voted for the proposal the second time than voted for it the first time?

(1) 10 (2) 15

(3) 20 (4) 25

(5) None of the above

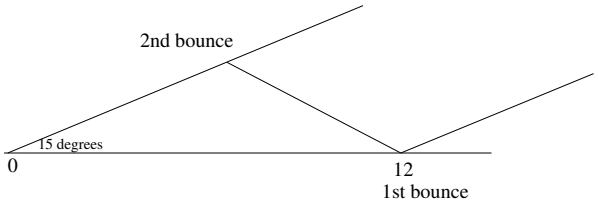
28. A certain right triangle has its area numerically equal to its perimeter. The length of each side is an even integer. What is the perimeter?

(1) 26 (2) 28

(3) 30 (4) 32

(5) None of the above

29. Two walls meet at a fifteen degree angle, as shown. An incoming particle enters parallel to one wall and bounces off the other wall 12 feet from the vertex. Assuming perfect bounces, how many feet from the vertex will the particle be on the 11th bounce?



(1) 1 (2) 2

(3) 4 (4) 8

(5) None of the above

30. The graph of

$$4x^2 - 24x + y^2 + 10y + 45 = 0$$

is an ellipse. What is the average of the lengths of the major and minor axes of this ellipse?

(1) 5 (2) 5.5

(3) 6 (4) 6.5

(5) None of the above

31. What is the ratio of the volume of a cube circumscribed about a sphere to the volume of a cube inscribed inside the same sphere?

(1) $\sqrt{2}$ (2) $\sqrt{3}$

(3) $2\sqrt{2}$ (4) $3\sqrt{3}$

(5) None of the above

32. Assume that $a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$ is the remainder when $x^{19}+2x^{14}+3x^9+4x^4+5$ is divided by $x^5 - x^4 + x^3 - x^2 + x - 1$. What is $\sum_{i=0}^{4} a_i$?

(1) 15 (2) 16

(3) 17 (4) 18

(5) None of the above

Bonus Questions: Show all your work.

1. Show the work that you did for number 28 on this test.

2. For how many positive integers n is there a triangle with three positive acute angles and sides of lengths 10, 24, and n? Show your work.