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- 1. What is the least common multiple of 48, 72, 128 and 500?
 - (1) 12000 (2) 120000
 - (3) 14400 (4) 144000
 - (5) None of the above
- **2.** A store sells bicycles and tricycles. If they currently have an inventory of 34 vehicles, and there are a total of 81 wheels, how many tricycles do they have?
 - (1) 9 (2) 10
 - (3) 11 (4) 12
 - (5) None of the above
- **3.** Expand

$$(x+1)(x+2)(x+3)(x+4),$$

and determine the coefficient on x^2 .

- (1) 35 (2) 36
- $(3) \quad 37 \qquad (4) \quad 38$
- (5) None of the above

4. If
$$f(x) = 3x - 2$$
 then $f^{-1}(4) = ?$

- (1) 2 (2) 10
- (3) -5/4 (4) 3/4
- (5) None of the above
- 5. There are 15 houses lining Riverview road. The average distance between the houses exceeds the average width of a house by 80 feet. The side-walk, which is 2920 feet long begins 30 feet before the first house and ends 30 feet past the last house. What is the average width of a house on Riverview road, in feet?
 - $(1) \quad 60 \quad (2) \quad 61$
 - $(3) \quad 62 \qquad \qquad (4) \quad 63$
 - (5) None of the above

- 6. The year my brother was born had 53 Sundays. My brother claims that he was born on a Wednesday. If today (March 13th) is his birthday, what do we know for sure?
 - (1) My brother is lying or mistaken.
 - (2) My brother must have been born in a leap year.
 - (3) My brother was actually born on a weekend.
 - (4) One can't tell anything from this little information.
 - (5) None of the above

7. The quantity
$$\left(\sqrt{4+\sqrt{4+\sqrt{4}}}\right)^4$$
 is equal to $(4+\sqrt{n})^2$

for some integer n. What is n?

- (1) 4 (2) 6
- (3) 8 (4) 10
- (5) None of the above
- 8. If the length of a side of a certain square is increased by 4 feet, then the area is increased by 56 square feet. If the length of a side of this bigger square is then increased by 4 feet, how many square feet bigger than the original square is this new square?
 - (1) 56 (2) 100
 - (3) 121 (4) 144
 - (5) None of the above
- **9.** I am teaching two sections of a calculus course this term. The section which meets at 8:00 has 30 students in it, and their class average on the first test was 40. My 9:00 class has 20 students in it, and their class average on the first test was 50. What is the average score on this test over the set of all of my students in this course this term?
 - (1) 44 (2) 45
 - (3) 46 (4) 47
 - (5) None of the above

10. What is the sum of the solutions to the equation

$$1 = \frac{6}{x} - \frac{8}{x(x+3)}?$$

$$(1) -2$$
 $(2) 3$

$$(3) -4$$
 $(4) 5$

(5) None of the above

11. Which quadrant is the vertex of the parabola

$$x - 2y - 2 = y^2$$

in? (Using the standard horizontal x and vertical y axes?)

(1) I	(2) II
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- (3) III (4) IV
- (5) None of the above
- **12.** The average of four different real numbers is 6. Which of the following *must* be true?
 - 1. Two of the numbers are less than 6 and two are greater than 6.
 - 2. If the smallest number is zero, then none of the numbers is greater than 12.
 - 3. At least one of the numbers is less than 6.
 - $(1) \quad 3 \text{ only} \qquad (2) \quad 2 \text{ only}$
 - (3) 2 and 3 only (4) 1, 2, and 3
 - (5) None of the above
- 13. What is the remainder when $x+x^{25}+x^{81}+x^9+x^{49}$ is divided by $-x+x^3$?
 - (1) x^{27} (2) $x^2 + 5x + 1$
 - (3) $5x^2$ (4) 5x
 - (5) None of the above
- 14. Which of the following numbers is irrational?
 - (1) $\frac{22}{7}$ (2) 3.141592
 - $(3) \quad 2.78181818\dots \qquad (4) \quad .112123123412345\dots$
 - (5) None of the above

15. In pictured triangle ABC, there is a right angle at A. If the measure of the angle at B is 22 degrees, and if M is the midpoint of the side \overline{BC} , what is the measure of angle AMC, in degrees?



- (1) Can't be determined (2) 22
- (3) 33 (4) 44
- (5) None of the above
- 16. Two vertical telephone poles are 50 feet apart. One is 20 feet tall, while the other is 60 feet tall. A blue laser shines from the top of the shorter pole to the base of the taller pole, and a red laser shines from the top of the taller pole to the base of the shorter pole. What is the height of the point where the laser beams meet?
 - (1) 15 (2) 18
 - (3) 20 (4) 21
 - (5) None of the above
- 17. What is the ones digit of the number $2^{201} + 3^{151}$?
 - (1) 1 (2) 3
 - $(3) \quad 6 \quad (4) \quad 7$
 - (5) None of the above

$$r = \frac{1}{\frac{2}{a} + \frac{3}{b}}$$

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is solved for a in terms of b and r, then a =

- (1) $\frac{2br}{b-3r}$ (2) $\frac{2}{r} \frac{2}{3}b$
- $(3) \quad \frac{2b}{3br-1} \qquad \qquad (4) \quad \frac{2r}{3b-r}$
- (5) None of the above

- **19.** Mark is given a 5 minute head start in a 1 mile race with Tom. If Tom can run at a speed of 5 miles per hour and Mark at 3 miles per hour, who will win the race and by how much time?
 - (1) Mark, by 3 minutes
 - (2) Tom, by 3 minutes
 - (3) Tom, by 8 minutes
 - (4) Tom, by 12 minutes
 - (5) None of the above
- **20.** If 1 y is used as an approximation to the value of $\frac{1}{1+y}$ where |y| < 1, then the ratio of the error made to the correct value is
 - (1) y (2) y^2
 - (3) $\frac{1}{1+y}$ (4) $\frac{y^2}{1+y}$
 - (5) None of the above
- **21.** What is true about the intersection of the curves $y = \frac{x^2 16}{x 4}$ and y = 2x?
 - (1) The x coordinate of the only intersection point is 4. (3)
 - (2) There are more than two point in the intersection.
 - (3) There are no points in the intersection.
 - (4) The intersection consists of exactly two points.
 - (5) None of the above
- 22. When Darby and Maddie leave their respective houses at the same time and walk toward each other, they meet in exactly 6 minutes. If Darby leaves after letting Maddie have a 3 minute head start, then they meet after Darby has walked for 4 minutes. How many minutes would it take Darby to walk from her house to Maddie's house?
 - (1) 15 (2) 16
 - (3) 17 (4) 18
 - (5) None of the above

23. Suppose that f and g are functions and that

$$f(g(x)) = x + 2$$

and that

$$f(s) = \frac{4-s}{s+1}$$

What is g(t)?

(1)
$$g(t) = f(t) + 2$$
 (2) $g(t) = f(t) - 2$

3)
$$g(t) = \frac{t+3}{2-t}$$
 (4) $g(t) = \frac{2-t}{t+3}$

- (5) None of the above
- 24. Find the sum of the coordinates of the midpoint of the line segment whose endpoints are the two intersection points of the curves

$$x^2 - 6x + y^2 = -8$$

and

$$y = x^2 - 4x + 4.$$

- (1) 1 (2) 2
- $(3) \ 3 \qquad (4) \ 4$
- (5) None of the above
- 25. Suppose that 100 people were surveyed about their opinions related to three sports played in January, and the following results were noted: 12 people liked both football and hockey, 10 liked both football and basketball, and 7 liked both basketball and hockey. There were 77 people who didn't like any of these sports, and there wasn't anybody who liked exactly one of these sports. How many liked all three?
 - (1) 1 (2) 2
 - (3) 3 (4) 4
 - (5) None of the above
- **26.** The 4-digit number 2abc is multiplied by 4 to get the 4-digit number cba2. What is a + b?
 - (1) 5 (2) 6
 - (3) 7 (4) 8
 - (5) None of the above

Junior Exam

27. The quadratic equation whose roots are reciprocals of the roots of

$$6x^2 - 11x + 4 = 0$$

is

- (1) $4x^2 11x + 6 = 0$ (2) $8x^2 10x + 3 = 0$
- (3) $4x^2 17x + 4 = 0$ (4) $\frac{1}{6}x^2 \frac{1}{11}x + \frac{1}{9} = 0$
- (5) None of the above
- **28.** At a certain party, everybody shakes hands with everybody else exactly once, although nobody (of course) shakes hands with themeselves. If there were a total of 55 handshakes, how many people were at the party?
 - (1) 10 (2) 11
 - (3) 12 (4) 13
 - (5) None of the above
- **29.** What is the largest value of $|x^2 16|$ if x satisfies $|x 4| \le .3$?
 - (1) 2.49 (2) 2.31
 - $(3) \quad 2.16 \qquad (4) \quad 5.34$
 - (5) None of the above
- **30.** Doug eats 1/3 of the M&M's which Mickey has "hidden" in the file cabinet. Then Tom sneaks in and eats 1/3 of what's left. Realizing she'd better act quickly, Mickey then eats 1/3 of what remains. When I go to check out the stash after that, I found that there are only 64 M&M's left. How many were there before Doug started eating?
 - (1) 200 (2) 213
 - (3) 219 (4) 221
 - (5) None of the above

Bonus Questions: Show all your work.

- 1. Generalize number 9 on this exam by finding the average score on the test over the set of all my students if my first class has n_1 students with class average of a_1 while my second class has n_2 students and a class average of a_2 .
- 2. Generalize number 15 on this exam by finding the height of the point where the laser beams meet if the first pole is a feet tall, the second pole is b feet tall, and the poles are c feet apart.

- **31.** Bozo the monkey has a set of four colored cups, which each can hold no more than one ball. Careful observation of Bozo leads one to note the following:
 - 1. If the red cup is empty, then the blue cup isn't empty.
 - 2. If the blue cup is empty, then the green and yellow cups have the same number of balls in them.
 - 3. If the green cup is empty, then the yellow cup isn't empty.

Suppose that you notice that the blue cup is empty. What is the sum of the number of balls in the four cups?

- $(1) \quad 0 \quad (2) \quad 1$
- (3) 2 (4) 3
- (5) None of the above
- **32.** If Tom can beat Kevin by 1/10 of a mile in a two mile race, and Kevin can beat Mark by 1/5 of a mile in a two mile race, then by how many miles would Tom beat Mark in a two mile race?
 - (1) .3 (2) .29
 - (3) .20 (4) .19
 - (5) None of the above