The Furman University Wylie Mathematics Tournament

Senior Examination
15 February 2003

Please provide the following information:

Name ______________________________

School ______________________________

Code __________

Instructions

1. Do not open this test booklet until instructed.
2. The test has 32 multiple-choice questions and two tie-breaker questions. You will have two hours to take the test.
3. Record your answers to the 32 multiple-choice questions on your answer sheet. You will be provided with color-coded sheets for your solutions to the tie-breaker questions.
4. When necessary, erase thoroughly on your answer sheet.
5. You will be penalized one-fourth of a point for each incorrect answer. You will not be penalized for any answer that is left blank. Your final score will be determined according to the formula:

$$\text{SCORE} = \#\text{CORRECT} - \frac{1}{4}\ \#\text{INCORRECT}$$

   Tie-breaker questions will only be graded to resolve ties.

6. The following notation and conventions will be observed:
   (a) The geometric figures are not necessarily drawn to scale.
   (b) $\triangle ABC$ is the triangle with vertices at A, B, and C.
   (c) $\angle ABC$ means the angle at vertex B of $\triangle ABC$.
   (d) AB denotes the segment with endpoints A and B.
   (e) Two points A and B on a circle separate the circle into two arcs. The arc with greater length is called the *major arc* AB; the arc with lesser length is called the *minor arc* AB.
   (f) Given a number x, $\lfloor x \rfloor$ denotes the greatest integer which is less than or equal to x. For example, $\lfloor \pi \rfloor = 3$ and $\lfloor -2.8 \rfloor = -3$.
   (g) We say that a ratio of two integers a/b is in *lowest terms* provided that a and b have no common divisor other than 1.

1. If the repeating decimal $.181818\cdots$ is expressed as a ratio of positive integers a/b in lowest terms, then $a+b$ is:

(1) 13$*$ (2) 117
(3) 118 (4) 39
(5) None of the above

2. If Q is the point on the circle

$$x^2 - 10x + y^2 + 6y + 29 = 0$$

which is furthest from the point $P(-1, -6)$, then the distance from P to Q is:

(1) $2\sqrt{5}$ (2) $2\sqrt{7}$
(3) $4\sqrt{5}*$ (4) $4\sqrt{7}$
(5) None of the above

3. If $x > 0$ and

$$\log(x) \geq \log(2) + \frac{1}{2}\log(x),$$

then:

(1) x has no minimum value;
(2) the minimum value of x is 1;
(3) the maximum value of x is 1;
(4) the minimum value of x is 4;*
(5) None of the above

4. If a farmer divides his herd of n cows among his four sons so that one son gets one-half of the herd, a second son gets one-third, a third son, one-eighth, and the fourth son, 11 cows, then n is:

(1) 288 (2) 216
(3) 240 (4) 264*
(5) None of the above

5. A segment of length 1 is cut, forming a greater and a lesser part. If the greater is to the whole as the lesser is to the greater, then the length of the greater part is:

(1) $(-1+\sqrt{5})/2$* (2) $(3-\sqrt{5})/2$
(3) $(-1+\sqrt{3})/2$ (4) $(\sqrt{3}-1)/2$
(5) None of the above

6. If $p(x) = ax^2 + bx + c$ leaves a remainder of 4 when divided by x, a remainder of 3 when divided by $x+1$, and a remainder of 1 when divided by $x-1$, then $p(2)$ is:

(1) -3 (2) -6*
(3) 3 (4) 6
(5) None of the above

7. If the sum of the solutions of the equation

$$\sin^2(x) - \sin(x) = \cos^2(x)$$

on the interval $[0, 2\pi]$ is expressed as $a\pi/b$, where a and b are positive integers, a/b in lowest terms, then $a+b$ is:

(1) 8 (2) 9*
(3) 10 (4) 11
(5) None of the above

8. Let

$$R(x) = \frac{x^3 - 2x^2 - 9x + 18}{x^2 - 4}.$$

Which of the following statements describes the graph of $y = R(x)$:

(1) the graph has two vertical asymptotes;
(2) the graph has two holes in it;
(3) the graph has one hole and one vertical asymptote
(4) the graph has neither holes nor asymptotes;
(5) None of the above

9. The number

$$\frac{1}{\sqrt{292} - \sqrt{290}}$$

is most nearly equal to which of the following integers:

(1) 15 (2) 16
(3) 17* (4) 18
(5) None of the above

10. If p is the smallest solution of $2^x + 15/2^x = 8$, then 4^p is:

(1) 9* (2) $\sqrt{3}$
(3) 3 (4) 25
(5) None of the above

11. If $P(x)$ is a polynomial with rational coefficients and roots at 0, 1, $\sqrt{2}$, and $1 - \sqrt{3}$, then the degree of $P(x)$ is at least

(1) 4 (2) 5
(3) 6* (4) 8
(5) None of the above

12. If $x \geq 0$, $y \geq 0$, $x + y \leq 3$, and $x + 7y \leq 7$, then the maximum value of $x + 2y$ is:

(1) 2 (2) 3
(3) 11/3* (4) 13/3
(5) None of the above

13. When a 6 foot man looks at the top of an obelisk, his line of sight makes an angle of 30° with the horizontal. When he moves 20 feet closer to the obelisk, his line of sight makes an angle of 45° with the horizontal. Rounded to the nearest foot, the approximate height of the obelisk is:

(1) 33* (2) 34
(3) 35 (4) 36
(5) None of the above

14. If a triangle has sides of length 3, 4 and 6, then the sine of the obtuse angle is:

(1) $\sqrt{455}/24$* (2) $\sqrt{3}/2$
(3) $\sqrt{255}/18$ (4) $\sqrt{113}/12$
(5) None of the above

15. The radius of the circle passing through the points $(.6, 50.2)$, $(42.6, 35.8)$, and $(-8.8, -15.6)$ is:

(1) 34 (2) 35
(3) 36 (4) 37*
(5) None of the above

16. The hands of a broken watch no longer move at the correct rates. Measured against a true clock, the "hour" hand makes a complete revolution in 1 hour and 20 minutes, and the "minute" hand makes a complete revolution in 1 hour and 4 minutes. If the hands are presently aligned, then the elapsed time, as measured by a true clock, until the next alignment is:

(1) 5 hours, 15 minutes (2) 5 hours, 20 minutes*
(3) 5 hours, 30 minutes (4) 5 hours, 45 minutes
(5) None of the above

17. At a certain ranch, the animals sell for whole numbers of dollars. If Tom bought as many goats as the price per goat, Kathy bought as many sheep as the price per sheep, and Tom spent $187.00 more than Kathy, then the smallest number of goats that Tom could have purchased is:

(1) 11 (2) 12
(3) 13 (4) 14*
(5) None of the above

18. Let I denote the set of all numbers m such that the line $y = mx$ does not intersect the parabola $y = x^2 + 1$. I is a bounded interval. The length of I is:

(1) 3 (2) 3.5
(3) 4* (4) 4.5
(5) None of the above

19. The sequence of rational numbers

$$1, \frac{7}{3}, \frac{29}{13}, \frac{123}{55}, \frac{521}{233}, \ldots$$

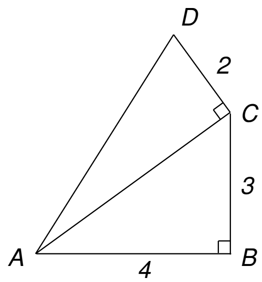
of the form a_n/b_n is generated by setting $a_1 = b_1 = 1$ and

$$\begin{cases} a_{n+1} = 2a_n + 5b_n \\ b_{n+1} = a_n + 2b_n \end{cases}$$

for $n \geq 1$. The limiting value of a_n/b_n is:

(1) $(1 + \sqrt{5})/2$ (2) $\sqrt{2}$
(3) $\sqrt{5}$* (4) $\sqrt{6}$
(5) None of the above

20. If the tangent of $\angle DAB$ is expressed as a ratio of positive integers a/b in lowest terms, then $a+b$ is:



(1) 31 (2) 33
(3) 35 (4) 37*
(5) None of the above

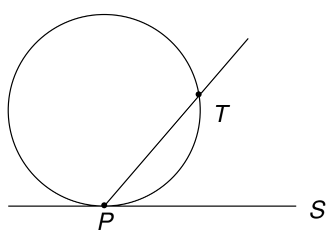
21. The coefficient of x in the expansion of

$$(1+x)(1+2x)(1+3x)\cdots(1+100x)$$

is:

(1) 4950 (2) 5000
(3) 5050* (4) 5100
(5) None of the above

22. In the figure below, the segment SP is tangent to the circle at P, and T is a point on the circle. If $\angle TPS$ is $12°$, then the fraction of the circumference comprised by the minor arc PT is:



(1) 1/30 (2) 1/24
(3) 1/15* (4) 1/10
(5) None of the above

23. The hundredths digit of $(1.1)^7 - (1.1)^6$ is:

(1) 6 (2) 7*
(3) 8 (4) 9
(5) None of the above

24. The number

$$\left(7+5\sqrt{2}\right)^{1/3} + \left(7-5\sqrt{2}\right)^{1/3}$$

is equal to:

(1) 2* (2) 3
(3) $2(7)^{1/3}$ (4) $(2+\sqrt{2})^{1/3}$
(5) None of the above

25. If

$$x = \sqrt{7+\sqrt{1+\sqrt{7+\sqrt{1+\sqrt{7+\cdots}}}}}$$

then $x^4 - 14x^2 - x$ is:

(1) 50 (2) -50
(3) 48 (4) -48*
(5) None of the above

26. The remainder when 3^{89} is divided by 7 is:

(1) 2 (2) 3
(3) 4 (4) 5*
(5) None of the above

27. If the sides of a triangle are 25, 39, and 40, then the diameter of the circumscribing circle is:

(1) 125/3* (2) 133/3
(3) 41 (4) 42
(5) None of the above

28. A farmer has a field containing three thin trees that form a right triangle with an area of 100 square yards. The farmer tethers three goats, X, Y, and Z, at the midpoints of the sides of the triangle, with goat Z at the midpoint of the hypotenuse. If each goat is tethered with just enough rope to reach the trees adjacent to its side, then the total area that can be reached by X or Y but not by Z is:

(1) 50 sq. yds. (2) 75 sq. yds.
(3) 100 sq. yds.* (4) 150 sq. yds.
(5) None of the above

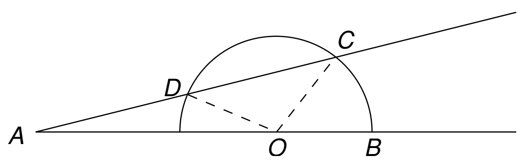
29. A segment AT is tangent to a circle at T. If the length ℓ of AT is four-thirds the length of the radius r of the circle, then the distance from A to the circle is:

(1) $\ell/2$* (2) $r/2$
(3) r (4) ℓ
(5) None of the above

30. Let a quadrilateral be inscribed within a circle, cutting the circle into four arcs. Let a point be selected from each arc, creating four points, say A, B, C, and D. To each of these points, let a triangle be formed by the point and the adjacent corners of the quadrilateral. Then the sum of the angles thus formed at A, B, C, and D is:

(1) 360° (2) 540°*
(3) 720° (4) 1080°
(5) None of the above

31. Let a protractor be laid upon an angle, as pictured below. If the measure if $\angle BOC$ is 47°, and the measure of $\angle BOD$ is 163°, then the measure of $\angle BAC$ is:



(1) 7.5° (2) 15°*
(3) 30° (4) 36°
(5) None of the above

32. The roots of $x^2 + bx + c = 0$ are both real and greater than 1. If $s = b + c + 1$, then s:

(1) may be less than zero;
(2) may be equal to zero;
(3) must be greater than zero;*
(4) must be less than zero;
(5) None of the above

Bonus Problem 1. Peter and Ryan each make a whole number of widgets per hour. Ryan is faster than Peter, and together they make less than 30 widgets per hour. If in 9 hours Peter can make 38 more widgets than Ryan can make in 5 hours, then how many widgets can Peter make in an hour?

Bonus Problem 2. A car moves around a circular track with uniform angular speed. An observer sits outside of the track at a point A, level with the track. If at a point B on the track, the car is pointing directly at A, at a point C on the track the car is pointing directly away from A, the measure of $\angle BAC$ is 168°, and the car moves from B to C in 20 seconds, then how long does it take the car to complete one lap of the track?