The Furman University Wylie Mathematics Tournament

Junior Examination
15 February 2003

Please provide the following information:

Name _____

School _____

Code _____

Instructions

1. Do not open this test booklet until instructed.
2. The test has 32 multiple-choice questions and two tie-breaker questions. You will have two hours to take the test.
3. Record your answers to the 32 multiple-choice questions on your answer sheet. You will be provided with color-coded sheets for your solutions to the tie-breaker questions.
4. When necessary, erase thoroughly on your answer sheet.
5. You will be penalized one-fourth of a point for each incorrect answer. You will not be penalized for any answer that is left blank. Your final score will be determined according to the formula:

$$\text{SCORE} = \#\text{CORRECT} - \frac{1}{4}\ \#\text{INCORRECT}$$

   Tie-breaker questions will only be graded to resolve ties.

6. The following notation and conventions will be observed:
   (a) The geometric figures are not necessarily drawn to scale.
   (b) $\triangle ABC$ is the triangle with vertices at A, B, and C.
   (c) $\angle ABC$ means the angle at vertex B of $\triangle ABC$.
   (d) AB denotes the segment with endpoints A and B.
   (e) Two points A and B on a circle separate the circle into two arcs. The arc with greater length is called the *major arc* AB; the arc with lesser length is called the *minor arc* AB.
   (f) Given a number x, $\lfloor x \rfloor$ denotes the greatest integer which is less than or equal to x. For example, $\lfloor \pi \rfloor = 3$ and $\lfloor -2.8 \rfloor = -3$.
   (g) We say that a ratio of two integers a/b is in *lowest terms* provided that a and b have no common divisor other than 1.

1. The value of $\left[2-3(2-3)^{-1}\right]^{-1}$ is:

(1) 5 (2) -5
(3) $1/5*$ (4) $-1/5$
(5) None of the above

2. The value of

$$1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4}}}$$

is:

(1) $43/30*$ (2) $35/24$
(3) $17/12$ (4) $5/4$
(5) None of the above

3. The expression

$$\frac{1/2-1/3}{1/5+1/8}$$

is equal to:

(1) $18/39$ (2) $19/39$
(3) $20/39*$ (4) $7/13$
(5) None of the above

4. The number

$$\frac{\sqrt{3}}{\sqrt{3}-\sqrt{2}}$$

is equal to:

(1) $(3-\sqrt{6})/5$ (2) $(3+\sqrt{6})/5$
(3) $3-\sqrt{6}$ (4) $3+\sqrt{6}*$
(5) None of the above

5. If a loaf of bread costs 80 cents plus a third of a loaf, then the number of whole loaves of bread that can be bought with 10 dollars is:

(1) 6 (2) 7
(3) $8*$ (4) 9
(5) None of the above

6. If 78 is divided into three parts proportional to one, one-third, and one-sixth, then the middle part is:

(1) $28/3$ (2) $52/3*$
(3) $55/3$ (4) 26
(5) None of the above

7. If a straight line joins the points $(-1,1)$ and $(3,9)$, then its x-intercept is:

(1) $-3/2*$ (2) $-2/3$
(3) $2/5$ (4) 2
(5) None of the above

8. If the repeating decimal $.181818\cdots$ is expressed as a ratio of positive integers a/b in lowest terms, then $a+b$ is:

(1) $13*$ (2) 39
(3) 117 (4) 118
(5) None of the above

9. If $2^x/(1+2^x)=1/4$, then $8^x/(1+8^x)$ is:

(1) $1/27$ (2) $1/28*$
(3) $3/4$ (4) $3/7$
(5) None of the above

10. If a line L is tangent to the circle $x^2+y^2=1$ at the point $(5/13,12/13)$, then the x-intercept of L is:

(1) $13/12$ (2) $12/13$
(3) $144/65$ (4) $13/5*$
(5) None of the above

11. The distance between the centers of the circles

$$x^2-8x+y^2-28y+112=0$$
$$x^2+6x+y^2+20y-12=0$$

is:

(1) $\sqrt{577}$ (2) $25*$
(3) 24 (4) $\sqrt{65}$
(5) None of the above

12. If a 6 foot man is 18 feet from a 30 foot streetlight, then the length of his shadow is:

(1) 3 ft. (2) 3.5 ft.
(3) 4 ft. (4) 4.5 ft.*
(5) None of the above

13. Through which quadrants does the circle

$$x^2 + 4x + y^2 - 6y + 1 = 0$$

pass?

(1) I, II, III* (2) II
(3) I, III, IV (4) I, II, III, IV
(5) None of the above

14. If two vertical poles are 20 feet and 80 feet high and 100 feet apart, then the height of the intersection of the lines joining the top of each pole to the foot of the opposite pole is:

(1) 10 ft. (2) 12 ft.
(3) 16 ft.* (4) 18
(5) None of the above

15. If a parabola has vertex $(0, 0)$ and focus $(1, 0)$, then the equation of the parabola is:

(1) $2y = x^2 - x$ (2) $x^2 = 4y$
(3) $y^2 = 4x*$ (4) $2x = y^2 - y$
(5) None of the above

16. If the sum of two numbers is 10 and their product is 20, then the sum of their reciprocals is:

(1) $1/2*$ (2) $1/10$
(3) 1 (4) 2
(5) None of the above

17. If a cubic polynomial $p(x)$ has roots at -1, 2, and 3, and if $p(0) = 1$, then the remainder when $p(x)$ is divided by $x - 1$ is:

(1) $2/3*$ (2) $3/2$
(3) $1/2$ (4) 2
(5) None of the above

18. The y-intercept of the line perpendicular to $4x + 9y = 13$ and passing through the center of the circle

$$x^2 - 2x + y^2 + 6y + 2 = 0$$

is:

(1) $-21/4*$ (2) $3/4$
(3) $-23/9$ (4) $31/9$
(5) None of the above

19. If 2 is a solution of the equation

$$x^3 + hx + 10 = 0,$$

then h equals:

(1) 10 (2) 9
(3) $-9*$ (4) -2
(5) None of the above

20. If a car travels from A to B at a rate of 36 miles per hour and returns at a rate of 64 miles per hour, then the average rate for the entire trip, rounded to the nearest mile per hour, is:

(1) 50 mph (2) 48 mph
(3) 47 mph (4) 46 mph*
(5) None of the above

21. The number of divisors of 360 is:

(1) 9 (2) 24*
(3) 6 (4) 12
(5) None of the above

22. The number of real solutions of the equation

$$\log_{\sqrt{3x^2-1}}(121) = 4$$

is:

(1) 0 (2) 1
(3) 2* (4) 4
(5) None of the above

23. Two classes took the same test. If one class of 20 students averaged 80%, and the other class of 30 students averaged 70%, then the average grade for both classes is:

(1) 77% (2) 75%
(3) 74%* (4) 72%
(5) None of the above

24. The number of solutions of the equation $4y + 15x = 181$ in positive integers is:

(1) 1 (2) 2
(3) 3* (4) 4
(5) None of the above

25. The number of distinct real solutions of the equation

$$4x^3 - 8x^2 + 5x - 1 = 0$$

is:

(1) 0 (2) 1
(3) 2* (4) 3
(5) None of the above

26. If a clock has minute and hour hands of length 6 and 5 inches respectively, then the distance between the tips of the hands at 4:00 is:

(1) $\sqrt{76}$ (2) $\sqrt{61 - 3\sqrt{3}}$
(3) $\sqrt{61 + 15\sqrt{3}}$ (4) $\sqrt{91}*$
(5) None of the above

27. If $\sin(x) = .6$ and $\pi/2 < x < \pi$, then $\sin(2x)$ is:

(1) .96 (2) $-.96$*
(3) .48 (4) $-.48$
(5) None of the above

28. If the distance from the origin to the line $24y + 7x = 168$ is expressed as a ratio of positive integers a/b in lowest terms, then $a + b$ is:

(1) 193$*$ (2) 194
(3) 195 (4) 196
(5) None of the above

29. The number of solutions of the equation

$$x^2 - y^2 = 63$$

in positive integers is:

(1) 2 (2) 3*
(3) 4 (4) 5
(5) None of the above

30. If

$$\frac{7x-2}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2}$$

for all x, when defined, then $B - A$ is:

(1) -1 (2) 0
(3) 1* (4) 2
(5) None of the above

31. If a square with an area of 40 is inscribed within a semi-circle, then the area of the square that can be inscribed within the full circle of the same radius is:

(1) 80 (2) 100*
(3) 120 (4) 160
(5) None of the above

32. Let A, B, and C be points on the perimeter of a circular field. If the length of AB is 13 rods, the length of BC is 37 rods, and the length of AC is 40 rods, then the diameter of the field is:

(1) 479/12 rods (2) 481/12 rods*
(3) 519/13 rods (4) 521/13 rods
(5) None of the above

Bonus Problem 1. Two bricklayers are employed to build a wall. Amos can build the wall in 10 hours; Bill can build the wall in 9 hours; when they work together, they can lay 10 fewer bricks per hour than their combined rates. If together it takes them 5 hours to build the wall, how many bricks are in the wall?

Bonus Problem 2. Given a pan balance and 1, 3, and 9 pound weights, how many different weights can be weighed, assuming that the objects to be weighed and the weights can be put into either pan of the scale?