The Furman University Wylie Mathematics Tournament

Ciphering Competition

Divison 2

The House Rules

1. Express numbers in their simplest form:
   (a) Combine integers, e.g., 10 instead of $7+3$.
   (b) Express fractions in lowest terms, e.g., $1/7$ instead of $2/14$.
   (c) Reduce radicals, e.g., $2\sqrt{3}$ instead of $\sqrt{12}$.
2. Express intervals in either set-builder or interval notation, e.g.,

$$\{x : \ 2 \leq x < 4\} \quad \text{or} \quad [2, 4)$$

Division 2

Round A

Division 2 Round A

1. Evaluate $(81)^{.17}(81)^{.08}$.

Division 2 Round A

1. Evaluate $(81)^{.17}(81)^{.08}$.

Answer. 3

2. Consider the plane figure given by the relation:

$$x^2 + y^2 - 10x - 12y + 12 = 0.$$

Through which quadrants does this figure pass?

2. Consider the plane figure given by the relation:

$$x^2 + y^2 - 10x - 12y + 12 = 0.$$

Through which quadrants does this figure pass?

Answer. I, II, and IV

3. If the base and height of a rectangle are increased by 10% and 20% respectively, what is the percent increase in the area of the rectangle?

3. If the base and height of a rectangle are increased by 10% and 20% respectively, what is the percent increase in the area of the rectangle?

Answer. 32%

4. A rectangular field is half as wide as it is long and completely enclosed by x yards of fencing. Express the area of the field in terms of x.

4. A rectangular field is half as wide as it is long and completely enclosed by x yards of fencing. Express the area of the field in terms of x.

Answer. $x^2/18$

5. A flask contains a solution which is p percent salt. A second flask has twice the volume but half the concentration of salt as the first. If the flasks are mixed, express the percentage of salt in the resulting mixture in terms of p.

5. A flask contains a solution which is p percent salt. A second flask has twice the volume but half the concentration of salt as the first. If the flasks are mixed, express the percentage of salt in the resulting mixture in terms of p.

Answer. $2p/3$

6. The average of a set of 50 numbers is 38. If the numbers 45 and 55 are removed from the set, what is the average of the remaining numbers, expressed as a decimal?

6. The average of a set of 50 numbers is 38. If the numbers 45 and 55 are removed from the set, what is the average of the remaining numbers, expressed as a decimal?

Answer. 37.5

7. Find the solution set of the inequality

$$\frac{1}{x^2+4x-12} \geq \frac{1}{20}.$$

Division 2 Round A

7. Find the solution set of the inequality

$$\frac{1}{x^2+4x-12} \geq \frac{1}{20}.$$

Answer. $\{x|\ -8 \leq x < -6 \quad \text{or} \quad 2 < x \leq 4\}$ or $[-8,-6) \cup (2,4]$

8. Let $f(x) = x^4 - 4x^2 + 7$, let M be the minimum value of f, and let N be the number of times this minimum value is achieved. Find MN.

8. Let $f(x) = x^4 - 4x^2 + 7$, let M be the minimum value of f, and let N be the number of times this minimum value is achieved. Find MN.

Answer. 6

9. If $\sin(x) = 5/13$ and $\pi/2 < x < \pi$, what is $\sin(2x)$?

9. If $\sin(x) = 5/13$ and $\pi/2 < x < \pi$, what is $\sin(2x)$?

Answer. $-120/169$

Division 2

Round B

Division 2 Round B

1. If a pup is worth a pooch and a mutt; a pup and a pooch are worth a bird dog; and 2 bird dogs are worth 3 mutts, then how many pooches is a pup worth?

Division 2 Round B

1. If a pup is worth a pooch and a mutt; a pup and a pooch are worth a bird dog; and 2 bird dogs are worth 3 mutts, then how many pooches is a pup worth?

Answer. 5

Division 2 Round B

2. Let $H - 3 = vg$. If $H = 5$ when $g = 10$, what is H when $g = 30$?

2. Let $H - 3 = vg$. If $H = 5$ when $g = 10$, what is H when $g = 30$?

Answer. 9

3. Evaluate $\log_{16}\left(\sqrt[3]{.25}\right)$.

3. Evaluate $\log_{16}\left(\sqrt[3]{.25}\right)$.

Answer. $-1/6$

4. Evaluate the product

$$\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\cdots\left(1-\frac{1}{100}\right)$$

4. Evaluate the product

$$\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\cdots\left(1-\frac{1}{100}\right)$$

Answer. $1/50$

5. What are A and B if

$$\frac{x}{x^2+x-20} = \frac{A}{x+5} + \frac{B}{x-4}$$

for all x, when defined?

5. What are A and B if

$$\frac{x}{x^2+x-20} = \frac{A}{x+5} + \frac{B}{x-4}$$

for all x, when defined?

Answer. $A = 5/9$ and $B = 4/9$

6. Let A be a set of 10 elements and let B be a subset of A having 6 elements. How many subsets of A contain B?

6. Let A be a set of 10 elements and let B be a subset of A having 6 elements. How many subsets of A contain B?

Answer. 16

Division 2 Round B

7. The domain of $f(x) = \sqrt{1 - x - x^2}$ can be expressed as in interval. What is the length of this interval?

7. The domain of $f(x) = \sqrt{1 - x - x^2}$ can be expressed as in interval. What is the length of this interval?

Answer. $\sqrt{5}$

8. If the perimeter and the diagonal of a rectangle have length 82 and 29 respectively, what is the area of the rectangle?

8. If the perimeter and the diagonal of a rectangle have length 82 and 29 respectively, what is the area of the rectangle?

Answer. 420

9. Let p be the perimeter of an equilateral triangle inscribed within a circle. Express the area of the circle in terms of p.

9. Let p be the perimeter of an equilateral triangle inscribed within a circle. Express the area of the circle in terms of p.

Answer. $\pi p^2/27$

Division 2

Round C

Division 2 Round C

1. Evaluate $\log_2(.0625)$.

Division 2 Round C

1. Evaluate $\log_2(.0625)$.

Answer. -4

2. If the edge of a cube is increased by 50%, what is the percent increase in its surface area?

2. If the edge of a cube is increased by 50%, what is the percent increase in its surface area?

Answer. 125%

3. Find the area bounded by the x-axis, the y-axis, and the line $2x + 3y = 6$.

3. Find the area bounded by the x-axis, the y-axis, and the line $2x + 3y = 6$.

Answer. 3

4. Find the smallest positive real number x for which

$$4^{2\sin^2(x)}16^{\tan^2(x)}2^{4\cos^2(x)} = 256.$$

4. Find the smallest positive real number x for which

$$4^{2\sin^2(x)}16^{\tan^2(x)}2^{4\cos^2(x)} = 256.$$

Answer. $\pi/4$

5. Solve for x in the equation

$$x^2 + b^2 = (a - x)^2.$$

5. Solve for x in the equation

$$x^2 + b^2 = (a - x)^2.$$

Answer. $(a^2 - b^2)/(2a)$ or $(b^2 - a^2)/(-2a)$

6. List the solutions of $9x^4 = 37x^2 - 4$ in decreasing order.

6. List the solutions of $9x^4 = 37x^2 - 4$ in decreasing order.

Answer. $x = 2, 1/3, -1/3, -2$

7. Find the largest integer n such that 6^n divides $40!$.

7. Find the largest integer n such that 6^n divides 40!.

Answer. 18

8. The system of equations

$$2x + 3y = c$$
$$3x + dy = 5$$

has infinitely many solutions for x and y. Find d and c.

8. The system of equations

$$2x + 3y = c$$
$$3x + dy = 5$$

has infinitely many solutions for x and y. Find d and c.

Answer. $d = 9/2$ and $c = 10/3$

9. Let $O(0,0)$, $P(2,4)$, and $Q(a,b)$ be points on the curve $y = x^2$. If $\angle QOP$ is a right angle, what are a and b?

9. Let $O(0,0)$, $P(2,4)$, and $Q(a,b)$ be points on the curve $y = x^2$. If $\angle QOP$ is a right angle, what are a and b?

Answer. $a = -1/2$ and $b = 1/4$

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Ciphering Competition

Divison 1

The House Rules

1. Express numbers in their simplest form:
   (a) Combine integers, e.g., 10 instead of $7+3$.
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   (c) Reduce radicals, e.g., $2\sqrt{3}$ instead of $\sqrt{12}$.
2. Express intervals in either set-builder or interval notation, e.g.,

$$\{x : \ 2 \leq x < 4\} \quad \text{or} \quad [2, 4)$$

Division 1

Round A

Division 1 Round A

1. A long level highway bridge passes over a railroad track which is 100 feet below it and at right angles to it. If at a given instant the tip of a car going 66 feet per second is directly above the tip of a train going 88 feet per second, how far apart are their tips 10 seconds later?

Division 1 Round A

1. A long level highway bridge passes over a railroad track which is 100 feet below it and at right angles to it. If at a given instant the tip of a car going 66 feet per second is directly above the tip of a train going 88 feet per second, how far apart are their tips 10 seconds later?

Answer. $100\sqrt{122}$ feet

2. Evaluate the sum

$$\frac{3}{2}+\frac{5}{4}+\frac{9}{8}+\frac{17}{16}+\frac{33}{32}+\frac{65}{64}+\frac{129}{128}.$$

2. Evaluate the sum

$$\frac{3}{2}+\frac{5}{4}+\frac{9}{8}+\frac{17}{16}+\frac{33}{32}+\frac{65}{64}+\frac{129}{128}.$$

Answer. 1023/128

3. Evaluate

$$\cot\left[\sin^{-1}\left(\frac{8}{17}\right)\right].$$

3. Evaluate

$$\cot\left[\sin^{-1}\left(\frac{8}{17}\right)\right].$$

Answer. 15/8

4. Find x and y that satisfy the system of equations

$$\frac{2}{x} + \frac{3}{y} = \frac{17}{2}$$
$$\frac{1}{x} + \frac{2}{y} = 5$$

4. Find x and y that satisfy the system of equations

$$\frac{2}{x} + \frac{3}{y} = \frac{17}{2}$$
$$\frac{1}{x} + \frac{2}{y} = 5$$

Answer. $x = 1/2$ and $y = 2/3$

5. A triangle has sides of length 5, 12 and 13. What is the distance from the center of the incircle to the vertex of the triangle which is furthest from this center?

5. A triangle has sides of length 5, 12 and 13. What is the distance from the center of the incircle to the vertex of the triangle which is furthest from this center?

Answer. $2\sqrt{26}$

6. A set S contains 6 numbers. The average of each combination of size three from S is computed, resulting in the set of averages

$$2, 4, 6, \ldots, 38, 40.$$

What is the average value of the elements of S?

6. A set S contains 6 numbers. The average of each combination of size three from S is computed, resulting in the set of averages

$$2, 4, 6, \ldots, 38, 40.$$

What is the average value of the elements of S?

Answer. 21

7. If $\sin(x) = 3/5$ and $\sin(2x) = -24/25$, then $\tan(x)$ is:

7. If $\sin(x) = 3/5$ and $\sin(2x) = -24/25$, then $\tan(x)$ is:

Answer. $-3/4$

8. Find the solution set of the inequality

$$x^3 + x^2 - x - 1 < 0.$$

8. Find the solution set of the inequality

$$x^3 + x^2 - x - 1 < 0.$$

Answer. $\{x : x < -1, -1 < x < 1\}$ or $(-\infty, -1) \cup (-1, 1)$

9. If

$$2f(x+1) - 3f(2-x) = x^2$$

for all real numbers x, what is $f(2)$?

9. If

$$2f(x+1) - 3f(2-x) = x^2$$

for all real numbers x, what is $f(2)$?

Answer. $-2/5$

Division 1

Round B

Division 1 Round B

1. In base b, 68 is expressed as 125. What is 68 in base $b+1$?

Division 1 Round B

1. In base b, 68 is expressed as 125. What is 68 in base $b+1$?

Answer. 104

2. A belt is wrapped around two wheels. The larger wheel has a radius of 8 feet, and the smaller wheel has a radius of 6 feet. If the larger wheel makes 21 revolutions per second, then how many revolutions per second does the smaller wheel make?

2. A belt is wrapped around two wheels. The larger wheel has a radius of 8 feet, and the smaller wheel has a radius of 6 feet. If the larger wheel makes 21 revolutions per second, then how many revolutions per second does the smaller wheel make?

Answer. 28

3. For real numbers a and b, let $a * b = a^b + b^a$, when defined. Solve for x in the equation $2 * x = 100$.

3. For real numbers a and b, let $a * b = a^b + b^a$, when defined. Solve for x in the equation $2 * x = 100$.

Answer. 6

4. Find the inverse function of

$$f(x) = \frac{x+3}{x-4}.$$

4. Find the inverse function of

$$f(x) = \frac{x+3}{x-4}.$$

Answer. $f^{-1}(x) = \dfrac{3+4x}{x-1} = \dfrac{-3-4x}{-x+1} = \text{etc.}$

5. Let two numbers be chosen at random and without replacement from the set of the first 10 prime numbers. What is the probability that their sum is prime?

5. Let two numbers be chosen at random and without replacement from the set of the first 10 prime numbers. What is the probability that their sum is prime?

Answer. 1/9

6. Find the sum of all solutions to the equation

$$\frac{1}{x-2} - \frac{1}{x+6} = \frac{2}{5}.$$

6. Find the sum of all solutions to the equation

$$\frac{1}{x-2} - \frac{1}{x+6} = \frac{2}{5}.$$

Answer. -4

7. If a parallelogram has an area of 21 and diagonals of length 6 and 14, what is the degree measure of the smaller of the two angles between the diagonals?

7. If a parallelogram has an area of 21 and diagonals of length 6 and 14, what is the degree measure of the smaller of the two angles between the diagonals?

Answer. 30°

8. When I leave for work at 8:00 a.m. and average 30 mph, I am 15 minutes late. When I leave for work at 8:00 a.m. and average 60 mph, I am 10 minutes early. What time does work begin?

8. When I leave for work at 8:00 a.m. and average 30 mph, I am 15 minutes late. When I leave for work at 8:00 a.m. and average 60 mph, I am 10 minutes early. What time does work begin?

Answer. 8:35 a.m.

9. Let $x_0 = 0$ and let

$$x_k = x_{k-1} + k$$

for each integer $k \geq 1$. What is x_{2003}?

9. Let $x_0 = 0$ and let

$$x_k = x_{k-1} + k$$

for each integer $k \geq 1$. What is x_{2003}?

Answer. 2007006

Division 1

Round C

Division 1 Round C

1. At the upper South Carolina state fair, I went to booth #1 and doubled my money and spent $30.00 on my daughter. At booth #2 I tripled my money and spent $54.00 on my wife. At booth #3 I quadrupled my money, spent $72.00 on myself, and went home with $48.00. How much money did I have at the start?

Division 1 Round C

1. At the upper South Carolina state fair, I went to booth #1 and doubled my money and spent $30.00 on my daughter. At booth #2 I tripled my money and spent $54.00 on my wife. At booth #3 I quadrupled my money, spent $72.00 on myself, and went home with $48.00. How much money did I have at the start?

Answer. $29.00

2. Find the sum of the base-10 digits of the number

$$\left(10^{10} + 3 \cdot 10^5 + 1\right)^2.$$

2. Find the sum of the base-10 digits of the number

$$\left(10^{10} + 3 \cdot 10^5 + 1\right)^2.$$

Answer. 16

3. Evaluate

$$\sec\left(-\frac{675\pi}{4}\right).$$

3. Evaluate

$$\sec\left(-\frac{675\pi}{4}\right).$$

Answer. $-\sqrt{2}$

4. A conical tank (vertex down) has a diameter 12 feet and height 12 feet. If the tank is partially filled with water, express the volume of the water in the tank as a function of its height h.

4. A conical tank (vertex down) has a diameter 12 feet and height 12 feet. If the tank is partially filled with water, express the volume of the water in the tank as a function of its height h.

Answer. $\pi h^3/12$

5. From a group of 5 girls and 6 boys, how many committees of size 3 can be formed if both genders must be represented on each committee?

5. From a group of 5 girls and 6 boys, how many committees of size 3 can be formed if both genders must be represented on each committee?

Answer. 135

6. What is the range of the function

$$f(x) = 2x^2 - 12x + 23$$

on the domain: $D = \{x : 1 \leq x \leq 7\}$?

6. What is the range of the function

$$f(x) = 2x^2 - 12x + 23$$

on the domain: $D = \{x : 1 \leq x \leq 7\}$?

Answer. $\{x : 5 \leq x \leq 37\}$ or $[5, 37]$

7. For real numbers a and b, let

$$a \star b = \frac{2}{ab(a+b)}.$$

Express $2 \star (2 \star c)$ as the ratio of two polynomials in c, written in descending order by power.

7. For real numbers a and b, let

$$a \star b = \frac{2}{ab(a+b)}.$$

Express $2 \star (2 \star c)$ as the ratio of two polynomials in c, written in descending order by power.

Answer. $(c^4 + 4c^3 + 4c^2)/(2c^2 + 4c + 1)$

8. The sum of the radii of two concentric circles is 48. If a chord of length 36 of the larger circle is cut into three equal pieces by the smaller circle, then the radius of the larger circle is:

8. The sum of the radii of two concentric circles is 48. If a chord of length 36 of the larger circle is cut into three equal pieces by the smaller circle, then the radius of the larger circle is:

Answer. 27

9. Find the solution set of the inequality:

$$|2x + 3| \leq 4 + |x|.$$

9. Find the solution set of the inequality:

$$|2x + 3| \leq 4 + |x|.$$

Answer. $\{x : -7 \leq x \leq 1\}$ or $[-7, 1]$