

1. How many Februarys since 1900 have had five Saturdays?

- (1) 4 (2) 5  
 (3) 6 (4) 7  
 (5) None of the above

2. The base  $b$  numbers 13, 42, and 101 are in arithmetic progression. Find  $b$ .

- (1) 6 (2) 7  
 (3) 8 (4) 9  
 (5) None of the above

3. Evaluate

$$\frac{1}{\log_4 6} + \frac{1}{\log_9 6}.$$

- (1) 1 (2) 2  
 (3) 3 (4) 4  
 (5) None of the above

4. In quadrilateral  $ABCD$ ,  $AB = 2$ ,  $BC = CD = 4$ ,  $DA = 5$ , and the opposite angles  $A$  and  $C$  are equal. Find the length of the diagonal  $BD$ .

- (1)  $\sqrt{6}$  (2)  $\sqrt{10}$   
 (3)  $2\sqrt{10}$  (4)  $2\sqrt{6}$   
 (5) None of the above

5. Find the sum

$$1 - 4 + 9 - 16 + \cdots - 100^2.$$

- (1)  $-5000$  (2)  $-4585$   
 (3)  $-5020$  (4)  $-5050$   
 (5) None of the above

6. How many real solutions does the following equation have?

$$\sqrt{1+x+\sqrt{x}} = \sqrt{x+\sqrt{x+5}}.$$

- (1) 0 (2) 1  
 (3) 2 (4) 3  
 (5) None of the above

7. Professor Ab Sentminded forgot to mail in his census form, so the census taker came to his house.

**Census Taker:** How many children do you have?

**Ab:** Three. Twin girls and one son, George.

**Census Taker:** Their ages?

**Ab:** I forget, but I do recall that the sum of their three ages is 13, and the product is the same as my age, which I've already given you.

**Census Taker:** I still need to know whether George is your oldest child.

How old is the Professor?

- (1) 80 (2) 63  
 (3) 40 (4) 36  
 (5) None of the above

8. What is the smallest positive integer  $k$  so that  $2^k$  leaves a remainder of 1 when divided by each of 5, 7, and 31?

- (1) 60 (2) 58  
 (3) 45 (4) 30  
 (5) None of the above

9. When Madison's dog chewed up her mathematics assignment, one particular equation was ripped apart. I found a piece of the beginning of the equation, and a piece of the end, but the middle was missing. The beginning piece was

$$x^5 - 9x^4 +$$

and the ending piece was

$$+11 = 0.$$

Fortunately, the teacher had promised that all the roots would be integers. How many times is  $-1$  a root?

- (1) 1 (2) 2  
 (3) 3 (4) 4  
 (5) None of the above

10. The nine members of the Travelers Rest city council (4 Republicans, 3 Democrats, 2 Independents) convene each Tuesday. How many handshakes are there if Republicans will not shake hands with Democrats, but all other possible handshakes between council members occur?

- (1) 12                                      (2) 16  
 (3) 20                                      (4) 24  
 (5) None of the above

11. A certain fraction  $r$  is represented in base  $b$  by

$$.111111\dots$$

while in base  $2b$  it takes the simpler form  $.2b$ . What is  $r$ ?

- (1)  $1/2$                                       (2)  $1/3$   
 (3)  $1/5$                                       (4)  $1/10$   
 (5) None of the above

12. Solve for  $x$  :

$$\log_2 3x = \log_3 2x.$$

- (1)  $1/2$                                       (2)  $1/3$   
 (3)  $1/5$                                       (4)  $1/6$   
 (5) None of the above

13. Solve for  $t$ .

$$4^{t+1} + 4^{t+2} + 4^{t+3} + 4^{t+4} = 170.$$

- (1)  $-1/4$                                       (2)  $-1/2$   
 (3)  $1/2$                                       (4)  $1/4$   
 (5) None of the above

14. Good old Professor Ab Sentminded was the only witness to the getaway portion of a bank robbery, and the state trooper had some questions.

**Trooper:** Did you see the license?

**Ab:** Yes, indeed. Factoring license plate numbers is my favorite pastime.

**Trooper:** Did you factor it?

**Ab:** Well, I was distracted by your siren and forgot the number, but before that I recall that I tried the prime divisors 2, 3, 5, 7, and 11, but got a remainder of 1 each time.

**Trooper:** Do you remember anything else?

**Ab:** Yes, come to think of it, the number had two even digits and two odd digits.

The trooper then knew the number. What was it?

- (1) 9461                                      (2) 9421  
 (3) 9241                                      (4) 9641  
 (5) None of the above

15. Equal length guy wires supporting two telephone poles are staked at the same point on a line between the poles where they meet at right angles. One reaches to a height of 10 meters, the other to a height of 11 meters. How far apart are the poles, in meters?

- (1) 7    (2) 12  
 (3) 18    (4) 23  
 (5) None of the above

16. In a certain lottery, three successive digits  $a, b, c$  are drawn. What is the probability that

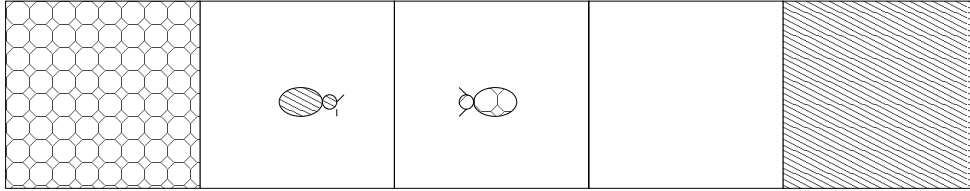
$$a > b > c?$$

- (1)  $1/25$                                       (2)  $2/25$   
 (3)  $3/25$                                       (4)  $4/25$   
 (5) None of the above





31. Two bugs spend their lives traveling between the five squares shown below. However, the spotted bug will not enter the striped square while the striped bug will not enter the spotted square. Otherwise, after pausing exactly one second in a square, a bug moves to an adjacent square. If it has a choice of directions it chooses left or right with equal probability. Given that the bugs start in the spots shown on the diagram, for what fraction of the bugs' lives are they together in a square?



- (1)  $1/8$                       (2)  $1/5$   
 (3)  $1/3$                       (4)  $1/4$   
 (5) None of the above
32. The following is a polynomial:

$$\sqrt[3]{x^9 - 3x^8 + 18x^7 - 28x^6 + 84x^5 - 42x^4 + 98x^3 + 72x^2 + 15x + 1}.$$

Find the sum of the squares of the coefficients of this polynomial.

- (1) 28                          (2) 30  
 (3) 32                          (4) 34  
 (5) None of the above

**Bonus Questions: Show all your work.**

The solution to #1 should be written on the yellow sheet labeled “#1”, and the solution to #2 should be written on the blue sheet labeled “#2.”

1. Prove that for every positive integer  $n$ , the number  $k = n(n + 1)(n + 2)$  is divisible by 6.
2. Use the result of the previous problem to prove that for every positive integer  $n$ , the number  $m = n(n^2 + 5)$  is divisible by 6.