- 1. How many Februarys since 1900 have had five Saturdays?
  - (1) 4 (2) 5
  - (3) 6 (4) 7
  - (5) None of the above
- 2. The base b numbers 13, 42, and 101 are in arithmetic progression. Find b.

(1)	6	(2) 7
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- (3) 8 (4) 9
- (5) None of the above
- **3.** Evaluate

$$\frac{1}{\log_4 6} + \frac{1}{\log_9 6}.$$

- (1) 1 (2) 2
- $(3) \quad 3 \quad (4) \quad 4$
- (5) None of the above
- 4. In quadrilateral ABCD, AB = 2, BC = CD = 4, DA = 5, and the opposite angles A and C are equal. Find the length of the diagonal BD.
  - (1)  $\sqrt{6}$  (2)  $\sqrt{10}$
  - (3)  $2\sqrt{10}$  (4)  $2\sqrt{6}$
  - (5) None of the above
- 5. Find the sum

$$1 - 4 + 9 - 16 + \dots - 100^2$$
.

- (1) -5000 (2) -4585
- (3) -5020 (4) -5050
- (5) None of the above
- 6. How many real solutions does the following equation have?

$$\sqrt{1 + x + \sqrt{x}} = \sqrt{x + \sqrt{x + 5}}$$

- $(1) \quad 0 \quad (2) \quad 1$
- (3) 2 (4) 3
- (5) None of the above

7. Professor Ab Sentminded forgot to mail in his census form, so the census taker came to his house.

**Census Taker:** How many children do you have?

- Ab: Three. Twin girls and one son, George.
- Census Taker: Their ages?
- **Ab:** I forget, but I do recall that the sum of their three ages is 13, and the product is the same as my age, which I've already given you.
- **Census Taker:** I still need to know whether George is your oldest child.

How old is the Professor?

- (1) 80 (2) 63
- (3) 40 (4) 36
- (5) None of the above
- 8. What is the smallest positive integer k so that  $2^k$  leaves a remainder of 1 when divided by each of 5, 7, and 31?
  - (1) 60 (2) 58
  - (3) 45 (4) 30
  - (5) None of the above
- **9.** When Madison's dog chewed up her mathematics assignment, one particular equation was ripped apart. I found a piece of the beginning of the equation, and a piece of the end, but the middle was missing. The beginning piece was

$$x^5 - 9x^4 +$$

and the ending piece was

$$+11 = 0.$$

Fortunately, the teacher had promised that all the roots would be integers. How many times is -1 a root?

- (1) 1 (2) 2
- (3) 3 (4) 4
- (5) None of the above

- 10. The nine members of the Travelers Rest city council (4 Republicans, 3 Democrats, 2 Independents) convene each Tuesday. How many handshakes are there if Republicans will not shake hands with Democrats, but all other possible handshakes between council members occur?
  - (1) 12 (2) 16
  - (3) 20 (4) 24
  - (5) None of the above
- **11.** A certain fraction r is represented in base b by

## $.1111111 \cdots$

while in base 2b it takes the simpler form .2b. What is r?

- (5) None of the above

**12.** Solve for x:

$$\log_2 3x = \log_3 2x.$$

- (1) 1/2 (2) 1/3
- (3) 1/5 (4) 1/6
- (5) None of the above
- **13.** Solve for t.

$$4^{t+1} + 4^{t+2} + 4^{t+3} + 4^{t+4} = 170.$$

- (1) -1/4 (2) -1/2
- (3) 1/2 (4) 1/4
- (5) None of the above

14. Good old Professor Ab Sentminded was the only witness to the getaway portion of a bank robbery, and the state trooper had some questions.

**Trooper:** Did you see the license?

- **Ab:** Yes, indeed. Factoring license plate numbers is my favorite pastime.
- Trooper: Did you factor it?
- **Ab:** Well, I was distracted by your siren and forgot the number, but before that I recall that I tried the prime divisors 2, 3, 5, 7, and 11, but got a remainder of 1 each time.

**Trooper:** Do you remember anything else?

**Ab:** Yes, come to think of it, the number had two even digits and two odd digits.

The trooper then knew the number. What was it?

- $(1) \quad 9461 \qquad \qquad (2) \quad 9421$
- $(3) \quad 9241 \qquad \qquad (4) \quad 9641$

(5) None of the above

- 15. Equal length guy wires supporting two telephone poles are staked at the same point on a line between the poles where they meet at right angles. One reaches to a height of 10 meters, the other to a height of 11 meters. How far apart are the poles, in meters?
  - (1) 7 (2) 12
  - (3) 18 (4) 23
  - (5) None of the above
- 16. In a certain lottery, three successive digits a, b, c are drawn. What is the probability that

a > b > c?

- (1) 1/25 (2) 2/25
- (3) 3/25 (4) 4/25
- (5) None of the above

- 17. Find the largest integer value of n so that  $3^n$  divides (25! + 26!).
  - (1) 10 (2) 11
  - (3) 12 (4) 13
  - (5) None of the above
- 18. At present the 10% alcohol, 90% gasoline mixture known as gasohol costs 5% more than pure gasoline. Suppose the price of gasoline doubles while the cost of producing alcohol remains fixed. How much less than gasoline will gasohol then cost?
  - (1) 2% (2) 2.5%
  - (3) 3% (4) 3.5%
  - (5) None of the above
- **19.** Suzan's cat got hold of the mail again. The bill was ripped, and all that she could find were portions of the bill. She found a portion that said

198 tulip bulbs at

which she recognized as the beginning of the bill, and a portion that said

cents each for a total of

which had a missing number before the word "cents", and she found a portion of a number which read

9.2

but Suzan could tell that the first and last digit of this number representing the total price (in dollars) was missing. If a is the number of cents that one tulip bulb costs, what is true about a?

- (a) Its digits add to 6.
- (b) It is congruent to 7 mod 10.
- (c) It is congruent to 0 mod 10.
- (d) Its digits add to 8.
- (e) None of the above

- **20.** During a science experiment, Hannah placed a metal cube with edge length 2 centimeters into a cylindrical pot which contained water to a depth of 2 centimeters. This caused the water to rise another centimeter. What is the volume of water in the pot?
  - (1)  $16/\pi$  (2)  $8/\pi$
  - (3) 8 (4) 16
  - (5) None of the above
- 21. Evaluate the 89 term product

 $(\tan 1^\circ)(\tan 2^\circ)(\tan 3^\circ)\cdots(\tan 89^\circ).$ 

- (1) 2/3 (2) 3/4
- (3) 1/2 (4) 7/8
- (5) None of the above
- **22.** For a certain positive integer n, 5n+16 and 8n+29 have a common factor bigger than 1. What is the common factor?
  - (1) 14 (2) 15
  - (3) 17 (4) 19
  - (5) None of the above
- **23.** How many positive integers n have the property that both n and n + 1001 are perfect squares?
  - (1) 2 (2) 3
  - (3) 4 (4) 5
  - (5) None of the above
- **24.** The complex numbers  $\alpha$  and  $\beta$  satisfy the relations

$$\alpha^2 = -1$$

and

$$\beta^2 = -1 - \beta.$$

How many distinct numbers are formed when we compute all possible products

 $\alpha^r\cdot\beta^s$ 

for positive integers r, s?

- (1) 4 (2) 6
- (3) 8 (4) 12
- (5) None of the above

25. The natural numbers are grouped as follows:

 $\{1\}, \{2,3\}, \{4,5,6\}, \{7,8,9,10\}, \cdots$ 

with n numbers in the nth group. Find the sum of the numbers in the 100th group.

- (1) 499,950 (2) 500,050
- (3) 501,050 (4) 502,050
- (5) None of the above
- **26.** What is the smallest value of c so that

$$\sin x \le c + \cos x$$

for all angles x?

- (2)  $\sqrt{2}$ (1) 1 (3)  $\sqrt{3}/2$ (4)  $\sqrt{3}$
- (5) None of the above
- 27. The design below is constructed by trisecting a diameter of a circle of area one and erecting four semicircles as shown. Find the area of the shaded region.



- (3) 1/6
- (5) None of the above

- **28.** Let  $A = 2^5 \cdot 5^2$ . Let B be the product of all positive integers which are divisors of A (including 1 and A itself). Find  $\log_A B$ .
  - (2) 7 (1) 6
  - (3) 8 (4) 9
  - (5) None of the above
- **29.** A square has vertices at (-2, -1), (-2, 5), (4, 5), (4, -1). Find the slope of the line through the origin which cuts the area of the square into halves.
  - (1) 1 (2) 1.25
  - (4) 2(3) 1.75
  - (5) None of the above
- **30.** Sarah and Eleasa often play together after school. Each leaves in the direction of the other's house at precisely 3:30 PM. When Sarah walks and Eleasa rides her bike, they meet at 3:40 PM. When Eleasa walks and Sarah rides, they meet at 3:45 PM. When they both walk, they don't meet until 3:54 PM. At what time in the afternoon do they meet when they both ride?
  - (1) 3:35 (2) 3:36
  - (3) 3:37 (4) 3:38
  - (5) None of the above

## Senior Exam

**31.** Two bugs spend their lives traveling between the five squares shown below. However, the spotted bug will not enter the striped square while the striped bug will not enter the spotted square. Otherwise, after pausing exactly one second in a square, a bug moves to an adjacent square. If it has a choice of directions it chooses left or right with equal probability. Given that the bugs start in the spots shown on the diagram, for what fraction of the bugs' lives are they together in a square?



- (1) 1/8 (2) 1/5
- (3) 1/3 (4) 1/4
- (5) None of the above
- **32.** The following is a polynomial:

$$\sqrt[3]{x^9 - 3x^8 + 18x^7 - 28x^6 + 84x^5 - 42x^4 + 98x^3 + 72x^2 + 15x + 1}$$

Find the sum of the squares of the coefficients of this polynomial.

- (1) 28 (2) 30
- $(3) \quad 32 \qquad \qquad (4) \quad 34$
- (5) None of the above

Bonus Questions: Show all your work.

The solution to #1 should be written on the yellow sheet labeled "#1", and the solution to #2 should be written on the blue sheet labeled "#2."

- **1.** Prove that for every positive integer n, the number k = n(n+1)(n+2) is divisible by 6.
- **2.** Use the result of the previous problem to prove that for every positive integer n, the number  $m = n(n^2 + 5)$  is divisible by 6.