1. The product of three consecutive positive integers is 40 times their sum. Find the sum of the three numbers.
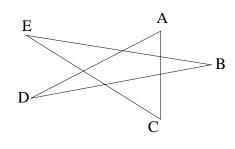
(1) 30   (2) 31

(3) 32   (4) 33

(5) None of the above

2. The egg race at Darby's birthday party was run as follows: 10 eggs are placed in a straight line with a basket. The first egg is 5 feet away from the basket, and the rest are at 10 foot intervals between eggs. The participant starts at the basket and must run back and forth fetching one egg at a time. How many feet does the participant run altogether?

(1) 605   (2) 1000

(3) 1200   (4) 1210

(5) None of the above

3. I have only nickels and dimes in my pocket. There are b coins in all and they are worth c cents. How many nickels do I have in terms of b and c?

(1) $(10b + c)/5$ (2) $(10b - c)/5$

(3) $(c - 10b)/5$ (4) $(c + 10b)/5$

(5) None of the above

4. How many different real numbers x satisfy the equation:

$$(x-5)(x-3)(x+1)(x-1) = (x-1)(x+1)(x+3)(x+5)?$$

(1) 1   (2) 2

(3) 3   (4) 4

(5) None of the above

5. Eight coins are lying, tails up, on a table. A move consists of turning over any 5 coins. How many moves are necessary to get all eight heads showing?

(1) 3   (2) 4

(3) 5   (4) 6

(5) None of the above

6. Hannah, Rachel, Eliza and Jess were comparing their stamp collections. Hannah had more commemoratives than Rachel, but together, Rachel and Jess had more than the combined total of Hannah and Eliza. Remarkably, Hannah and Rachel's combined total of commemoratives exactly equaled the combined total of Eliza and Jess. List the collectors in order according to the sizes of their collections of commemoratives, greatest to least.

(a) Hannah, Rachel, Eliza, Jess

(b) Hannah, Eliza, Jess, Rachel

(c) Hannah, Jess, Eliza, Rachel

(d) Jess, Hannah, Eliza, Rachel

(e) None of the above

7. The difference of the squares of the digits of a two-digit number is 45. If the number formed by reversing the digits is subtracted from the original number, that difference is also 45. What is the original number?

(1) 54   (2) 27

(3) 45   (4) 72

(5) None of the above

8. A boater, rowing upstream, did not see his hat fall overboard. When he discovered the loss twenty minutes later, he turned around and finally caught up with his hat, which had by then drifted two miles with the current. What is the speed of the current, in miles per hour?

(1) 2   (2) 3

(3) 4   (4) 5

(5) None of the above

9. What is the sum of the angles A, B, C, D, and E in the following figure, measured in degrees?
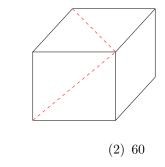

  (1) 150   (2) 165

  (3) 180   (4) 195

  (5) None of the above

10. Let $A = 2002 + \frac{1}{2002}$, $B = 2002 + \frac{1}{2002 + \frac{1}{2002}}$, and $C = 2002 + \frac{1}{2002 + \frac{1}{2002 + \frac{1}{2002}}}$. Which of the following is true?

  (1) $B < A < C$ (2) $B < C < A$

  (3) $C < B < A$ (4) $C < A < B$

  (5) None of the above

11. If

$$\begin{aligned} x + 2y + 3z &= 3 \\ y + 2z + 3x &= 2 \\ z + 2x + 3y &= 1, \end{aligned}$$

then what is the value of $z - x + 3y$?

  (1) 1   (2) 2

  (3) 3   (4) 4

  (5) None of the above

12. In the year 1980 I could make the claim that my age was the same as the last two digits of the year of my birth. Amazingly, my grandmother could make the same claim about herself. How old did my grandmother turn in the year I was born?

  (1) 45   (2) 50

  (3) 60   (4) 65

  (5) None of the above

13. Darby and Taylor had the same grade on the last calculus quiz and, for both of them, it was their highest quiz score this semester. It brought Darby's quiz average from 83 to 86 and Taylor's from 88 to 90. How many quizzes has each student in the class taken?

  (1) 4   (2) 5

  (3) 6   (4) 7

  (5) None of the above

14. At present the 10% alcohol, 90% gasoline mixture known as gasohol costs 5% more than pure gasoline. Suppose the price of gasoline doubles while the cost of producing alcohol remains fixed. How much less than gasoline will gasohol then cost?

  (1) 2%   (2) 2.5%

  (3) 3%   (4) 3.5%

  (5) None of the above

15. What is the measure of the angle between the two dotted lines shown on the cube below, measured in degrees? Note that one dotted line is on the front face of the cube, while the other is on the top face.

  (1) 45   (2) 60

  (3) 75   (4) 90

  (5) None of the above

16. What is the area of the triangle with vertices at $(0,0)$, $(3,1)$, and $(-1,3)$?

  (1) 4   (2) $4\sqrt{3}$

  (3) $4\sqrt{5}$ (4) 5

  (5) None of the above

17. What is the largest positive integer n so that both n and $n + 1001$ are perfect squares?

(1) 1024

(2) 1600

(3) 4624

(4) 250,000

(5) None of the above

18. The nine members of the Travelers Rest city council (4 Republicans, 3 Democrats, 2 Independents) convene each Tuesday. How many handshakes are there if Republicans will not shake hands with Democrats, but all other possible handshakes between council members occur?

(1) 12

(2) 16

(3) 20

(4) 24

(5) None of the above

19. Trains leave from Atlanta for Greenville every hour on the hour. The trip takes three hours. Each train waits at the Greenville depot one half hour and then returns to Atlanta. How many trains going the other way will a given train pass on its return trip?

(1) 3

(2) 4

(3) 5

(4) 6

(5) None of the above

20. A circle of radius 3 intersects a circle of radius 4 at points A and B. The angle between the tangents to the circles at point A is 90 degrees. Find the length of line segment AB.

(1) $\frac{24}{5}$

(2) $\frac{26}{5}$

(3) $\frac{28}{5}$

(4) $\frac{29}{5}$

(5) None of the above

21. The sum of three numbers in arithmetic progression is 12. The sum of their squares is 66. What is the difference between the 2nd and the 3rd number?

(1) 4

(2) 3

(3) 2

(4) 1

(5) None of the above

22. A square has vertices at $(-2, -1)$, $(-2, 5)$, $(4, 5)$, $(4, -1)$. Find the slope of the line through the origin which cuts the area of the square into halves.

(1) 1

(2) 1.25

(3) 1.75

(4) 2

(5) None of the above

23. What is the smallest positive integer k so that 2^k leaves a remainder of 1 when divided by each of 5, 7, and 31?

(1) 60

(2) 58

(3) 45

(4) 30

(5) None of the above

24. Solve for t.

$$4^{t+1} + 4^{t+2} + 4^{t+3} + 4^{t+4} = 170.$$

(1) $-1/4$

(2) $-1/2$

(3) $1/2$

(4) $1/4$

(5) None of the above

25. Professor Ab Sentminded forgot to mail in his census form, so the census taker came to his house.

> **Census Taker:** How many children do you have?
>
> **Ab:** Three. Twin girls and one son, George.
>
> **Census Taker:** Their ages?
>
> **Ab:** I forget, but I do recall that the sum of their three ages is 13, and the product is the same as my age, which I've already given you.
>
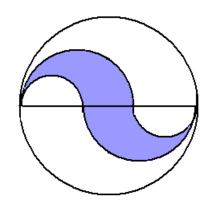> **Census Taker:** I still need to know whether George is your oldest child.

How old is the Professor?

(1) 80

(2) 63

(3) 40

(4) 36

(5) None of the above

26. How many real solutions does the following equation have?

$$\sqrt{1 + x + \sqrt{x}} = \sqrt{x + \sqrt{x + 5}}.$$

(1) 0

(2) 1

(3) 2

(4) 3

(5) None of the above

27. The design below is constructed by trisecting a diameter of a circle of area one and erecting four semicircles as shown. Find the area of the shaded region.

(1) 1/3

(2) 1/4

(3) 1/6

(4) 1/12

(5) None of the above

28. Sarah and Eleasa often play together after school. Each leaves in the direction of the other's house at precisely 3:30 PM. When Sarah walks and Eleasa rides her bike, they meet at 3:40 PM. When Eleasa walks and Sarah rides, they meet at 3:45 PM. When they both walk, they don't meet until 3:54 PM. At what time in the afternoon do they meet when they both ride?

(1) 3:35

(2) 3:36

(3) 3:37

(4) 3:38

(5) None of the above

29. What is the radius of the circle which passes through the points $(-5, 0)$, $(5, 0)$, and $(0, 1)$?

(1) 5

(2) 10

(3) 12

(4) 13

(5) None of the above

30. The roots of

$$x^2 - 30x + b = 0$$

are both positive. One root is the square of the other. If the two roots are r and s with $r > s$, what is $r - s + b$?

(1) 100

(2) 115

(3) 130

(4) 145

(5) None of the above

31. Good old Professor Ab Sentminded was the only witness to the getaway portion of a bank robbery, and the state trooper had some questions.

Trooper: Did you see the license?

Ab: Yes, indeed. Factoring license plate numbers is my favorite pastime.

Trooper: Did you factor it?

Ab: Well, I was distracted by your siren and forgot the number, but before that I recall that I tried the prime divisors 2, 3, 5, 7, and 11, but got a remainder of 1 each time.

Trooper: Do you remember anything else?

Ab: Yes, come to think of it, the number had two even digits and two odd digits.

The trooper then knew the number. What was it?

(1) 9461

(2) 9421

(3) 9241

(4) 9641

(5) None of the above

32. How many Februarys since 1900 have had five Saturdays?

(1) 4

(2) 5

(3) 6

(4) 7

(5) None of the above

Bonus Questions: Show all your work.
The solution to #1 should be written on the pink sheet labeled "#1", and the solution to #2 should be written on the green sheet labeled "#2."

1. Prove that for every positive integer n, the number $k = n(n+1)(n+2)$ is divisible by 6.

2. Use the result of the previous problem to prove that for every positive integer n, the number $m = n(n^2 + 5)$ is divisible by 6.