Furman University
Wylie Mathematics Tournament
Ciphering 2002

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Cipher House Rules

- All answers are integers(!)

- All answers must be written in standard form. For example, 8 not 2^3, and 10, not $\binom{5}{2}$.

- If a question asks for a ratio, you should still report your answer as an integer. For example, if $c = 8$ and $d = 2$ and you are asked for the ratio of c to d, you should report your answer as 4, not "8 to 2" or "4 to 1".

Division II Round I Ciphering

Participants in Round I ciphering from Division II schools should now make their way to the front.

Division II Round I – Number 1

Petersburg Virginia is 360 miles away from Greenville. If I start my trip from Greenville to Petersburg and drive half way at 30 mph, how many miles per hour must I drive the rest of the way in order to make the total time for the trip 9 hours?

Division II Round I – Number 2

If

$$\sin(\pi/3 + x) + \sin(\pi/3 - x) = a\cos(x),$$

what is a^2?

Division II Round I – Number 3

If $f(x) = \frac{x+3}{x-2}$, and $g(x)$ is such that $g(f(x)) = x$, then what is the value of $4 \cdot g(5)$?

Division II Round I – Number 4

If the y–coordinate of the vertex of

$$y = 2 + 3x - x^2$$

is written in the reduced form a/b, what is a?

Division II Round I – Number 5

How many 3-digit natural numbers are there which are multiples of 7?

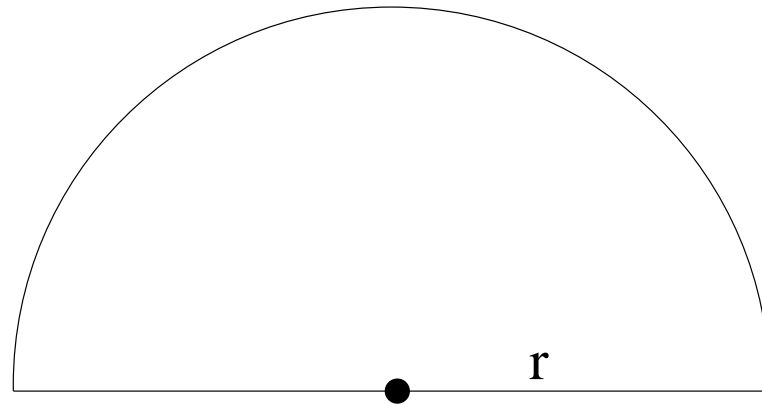# Division II Round I – Number 6

Find the sum of the first 40 positive odd integers subtracted from the sum of the first 40 positive even integers.

Division II Round I – Number 7

In the following diagram, the perimeter in feet is equal to the area in square feet.

The radius r of the semicircle has the form $r = \frac{k(2+\pi)}{\pi}$. What is k?

Division II Round II Ciphering

Participants in Round II ciphering from Division II schools should now make their way to the front.

Division II Round II – Number 1

Madison, Hannah and Darby are having a sleep-over in a room with a bag of candy bars. At 1:00, Madison woke up and ate 1/3 of the candy bars. At 2:00, Darby woke up and ate 1/3 of the remaining candy bars. At 3:00, Hannah woke up and ate 1/3 of what was left. In the morning, I counted 16 candy bars left in the bag. How many candy bars were in the bag when they went to bed?

Division II Round II – Number 2

Sales tax in Pickens County is 6%. If a purchase totals $445.73 with tax, then the amount of tax paid on the purchase is equal to 25 dollars and x cents. What is x?
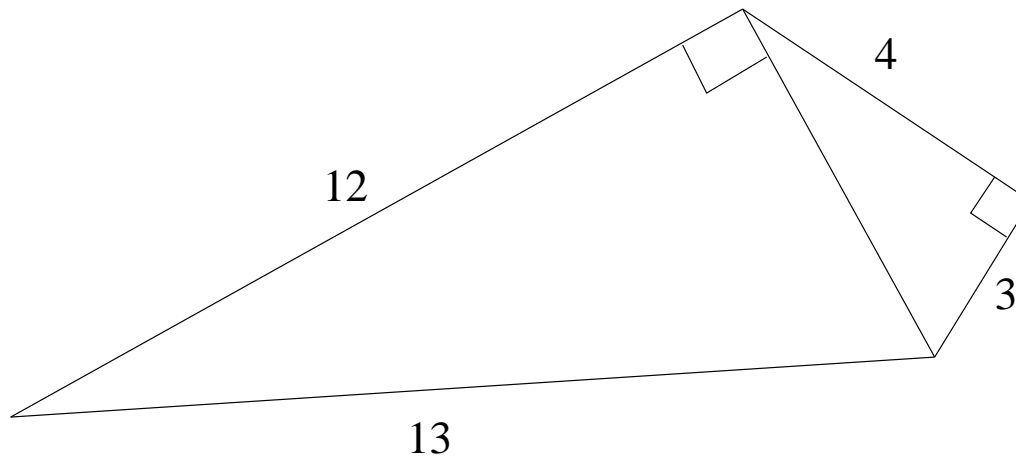
Division II Round II – Number 3

The probability of getting 70% or better on a true/false exam by simply guessing is $k/2^{10}$ for some integer k.
What is k?

Division II Round II – Number 4

What is the total enclosed area in the following figure?

Division II Round II – Number 5

Melvin invests $1\frac{1}{2}$ times as much in saving certificates as in government bonds. If his total investment amounts to $47,500, how many dollars does he invest in government bonds?

Division II Round II – Number 6

If $x < 0$, what is

$$\left| x + \sqrt{(x-2)^2} \right| \, ?$$

Division II Round II – Number 7

A cylindrical can with radius 12 and height 20 is full of water. If a sphere of radius 10 is lowered into the can and then removed, the volume of water which will remain in the can has the form $k\pi/3$ for some integer k. Find k.

Division II Round III Ciphering

Participants in Round III ciphering from Division II schools should now make their way to the front.

Division II Round III – Number 1

Find x if

$$\log_3 x + \log_9 x + \log_{27} x = 11$$

Division II Round III – Number 2

Compute $64 \cdot \sec^2 x$ if $\tan x = 5/8$ and $\sin x < 0$.

Division II Round III – Number 3

How many different positive integer factors of 42^5 are there?

Division II Round III – Number 4

What is the shortest distance between a point on

$$x^2 + y^2 - 14x - 4y + 44 = 0$$

and a point on

$$x^2 + y^2 + 10x + 6y + 18 = 0?$$

Division II Round III – Number 5

My daughter Hannah, who has a palindromic name, will turn a palindromic age during this palindromic year of 2002. Given that she is still a minor, what is the largest palindromic integer which is a proper divisor of the palindromic year of her birth?

Division II Round III – Number 6

Remember Professor Ab Sentminded from the written test? He *always* wears one black sock and one blue sock, which he calls "a pair." The professor's drawer contains 8 black socks and 8 blue socks, not in pairs. If Ab randomly selects individual socks (without replacement), how many socks *at most* would he have to pick before he is assured of having a pair to wear?

Division II Round III – Number 7

Find the sum of all real solutions to the equation

$$\sqrt{x + 13} - \sqrt{7 - x} = 2?$$

Division I Round I Ciphering

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Division I Round I – Number 1

If x is a positive number and the reciprocal of $x + 1$ is $x - 1$, then what is x^2?

Division I Round I – Number 2

If $\log_2\left((\log_{16} 2)^{\log_5(125)}\right) = -a$, then what integer is a?

Division I Round I – Number 3

If $x + y = 1$ and $x^2 - 1 = 2$, then what integer represents the value of $x^3 + yx^2 - x - y$?

Division I Round I – Number 4

If $(4, 2)$ is the midpoint of the line joining $(x, 4)$ and $(3, y)$, find $x + y$.

Division I Round I – Number 5

How many times does the graph of

$$f(x) = x^3 - x^2 + 2x + 4$$

cross the x axis?

Division I Round I – Number 6

The sides of a right triangle are a, $a + d$ and $a + 2d$, where a and d are positive. What is the ratio of a to d?

Division I Round I – Number 7

If

$$\log_{10} x = .4871 + 3$$

and

$$\log_{10} 3.07 = .4871,$$

what is x?

Division I Round II Ciphering

Participants in Round II ciphering from Division I schools should now make their way to the front.

Division I Round II – Number 1

A goat is tied in a grassy field by a 6 foot long rope to the outside corner of a shed measuring 4 feet by 5 feet. The area in square feet that the goat can graze has the form $a\pi/4$. What is a?

If θ is the sum of the coordinates of the center of

$$16x^2 + 16y^2 + 8x - 32y - 127 = 0,$$

Then what is $4 \cdot \theta$?

Division I Round II – Number 3

Find the sum of all real solutions to the equation

$$\log_{10}(x^2 - 3x + 6) = 1.$$

Division I Round II – Number 4

If $\sin\theta = 2/\sqrt{5}$, and θ is a central angle whose terminal side is in the first quadrant, then what is $5 \cdot \cos(2\theta)$?

Division I Round II – Number 5

If we write $30!$ as $2^n k$ where k is odd, find n.

Division I Round II – Number 6

The angles in a triangle are in the ratio 2:3:4. What is the size of the largest angle, in degrees?

Division I Round II – Number 7

What is the last digit of the number 7^{2002}?

Division I Round III Ciphering

Participants in Round III ciphering from Division I schools should now make their way to the front.

Division I Round III – Number 1

The average age of the set of mathematicians in a certain room is 50, while the average age of the computer scientists in the same room 35. The average age of everyone in the room is 40. What is the ratio of the number of computer scientists to the number of mathematicians, if everyone in the room is either a mathematician or a computer scientist, but no one is both.

Division I Round III – Number 2

The slope of the line between the points $(2, a)$ and $(|a|, -4)$ is 3. What is the reciprocal of the sum of all such possible values of a?

Division I Round III – Number 3

$$\frac{3}{2} + \frac{5}{4} + \frac{9}{8} + \frac{17}{16} + \frac{33}{32} + \frac{65}{64} - 7$$

is equal to

$$\frac{-k}{64}$$

for some integer k. What is k?

Division I Round III – Number 4

If $7^t = 3.4$, what is $\lfloor 7^{t+2} \rfloor$?

Division I Round III – Number 5

If $.2\overline{414141}$ is expressed as a rational number in reduced form, then the value of the numerator is:

Division I Round III – Number 6

How many real solutions does the following equation have on the interval $[0, 2\pi]$?

$$2\sin^2(n/2) + \cos^2(n) = 1$$

Division I Round III – Number 7

Dave and Dan working together can complete a certain job in 10 days. They began this job in early December, but at the end of the seventh day, Dan got sick and Dave, working alone, finished the job in 5 days. How long would it have taken Dan to do the job by himself?

That's All, Folks

Awards Ceremony to follow soon. Please be patient while we tally the results.