**1.** The sum of the series

$$1 - \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} - \frac{1}{32} + \frac{1}{64} - \frac{1}{128} - \cdots$$

is:

(a) 0 (b) $\star\frac{2}{7}$

(c) $\frac{6}{7}$ (d) $\frac{9}{32}$

(e) None of the above

**2.** How many of the numbers between 100 and 199 don't have exactly two identical digits?

(a) $\star$73 (b) 74

(c) 75 (d) 76

(e) None of the above

**3.** Bill belongs to a committee which consists of 4 females and 3 males, including him. A subcommittee of 3 members is to be selected, and the subcommittee must contain both sexes. In how many ways can the subcommittee be selected, if Bill is to be on the subcommittee?

(a) 5 (b) 7

(c) 9 (d) 11

(e) None of the above

**4.** The parallel sides of a trapezoid are 3 and 9. The non-parallel sides are 4 and 6. A line parallel to the bases divides the trapezoid into two trapezoids of equal perimeters. The ratio in which each of the non-parallel sides is divided is:

(a) 4:3 (b) 3:2

(c) $\star$4:1 (d) 3:1

(e) None of the above

**5.** A number is called *palindromic* if its digits read the same forward as backward. What is the largest integer $k$ so that it is true to say that all 4-digit palindromic numbers are divisible by $k$?

(a) 8 (b) 9

(c) 10 (d) $\star$11

(e) None of the above

**6.** If $f$ is a polynomial function of degree three with roots at 1, 2, and 3, then a valid statement concerning $f$ is:

(a) $f(0) \cdot f(4) = 0$ (b) $f(0) \cdot f(4) > 0$

(c) $\star f(0) \cdot f(4) < 0$ (d) $f(0) + f(4)$ doesn't exist

(e) None of the above

**7.** A very skinny 5 foot long fishing pole is to fit in a box without any bending. The box has dimensions $2 \times 2 \times n$. What is the smallest possible value for $n$?

(a) $\star\sqrt{17}$ (b) $\sqrt{18}$

(c) $\sqrt{19}$ (d) $\sqrt{20}$

(e) None of the above

**8.** What is the sum of the positive prime factors of the number 23,595? Repeated prime factors should be repeated in the sum.

(a) $\star$43 (b) 45

(c) 47 (d) 49

(e) None of the above

**9.** Given $f(x,y) = \frac{x^2+y^2}{x^2-y^2}$, which of the following is a true statement:

(a) $\star f(\frac{1}{x}, \frac{1}{y}) = -f(x,y)$ (b) $f(\frac{1}{x}, \frac{1}{y}) = f(x,y)$

(c) $f(\frac{1}{x}, y) = f(x,y)$ (d) $f(x, \frac{1}{y}) = f(x,y)$

(e) None of the above

**10.** A boat is tied to a dock by means of a cable which is 60 meters long. If the dock is 20 meters above the water and if the cable is being drawn in at the rate of 10 meters per minute, express the distance $y$ of the boat from the foot of the dock (in meters) after $t$ minutes.

(a) $\sqrt{100t^2 - 400}$ (b) $10\sqrt{t^2 - 40}$

(c) $40 - 10t$ (d) $\star 10\sqrt{t^2 - 12t + 32}$

(e) None of the above

**11.** Person $A$ can do a piece of work in 10 days. After he has worked for 2 days, $B$ comes and joins him and together they finish the work in 3 more days. In how many days could $B$ have done the work by himself?

(a) $\star$6 (b) 7

(c) 8 (d) 9

(e) None of the above

**12.** Two pipes together can fill a reservoir in 6 hours and 40 minutes. Find the the number of hours that the smaller pipe will take to fill the reservoir if one of the pipes can fill it in 3 hours less time than the other, working alone.

(a) $\star 15$ (b) 14

(c) 13 (d) 12

(e) None of the above

**13.** The equation $4x^2 - 8kx + 9 = 0$ has roots whose difference is 4. Which of the following is a possible value of $k$?

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$

(c) $\star\frac{5}{2}$ (d) $\frac{7}{2}$

(e) None of the above

**14.** Find the sum of all real solutions to

$$\frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}} = 3$$

(a) $\star\frac{5}{3}$ (b) 0

(c) 6.5 (d) $\frac{3+\sqrt{3}}{2}$

(e) None of the above

**15.** The equation $3y^2 + y + 4 = 2(6x^2 + y + 2)$ where $y = 2x$ is satisfied by:

(a) no value of $x$

(b) all values of $x$

(c) $\star x = 0$ only

(d) all integral values of $x$ only

(e) None of the above

**16.** A set of $n$ numbers has the sum of $s$. Each number of the set is increased by 20, then multiplied by 5, and then decreased by 20. The sum of the numbers in the new set thus obtained is:

(a) $s + 20n$ (b) $\star 5s + 80n$

(c) $s$ (d) $5s$

(e) None of the above

**17.** A square, with an area of 40, is inscribed in a semi-circle. The area of a square that could be inscribed in the entire circle with the same radius is:

(a) 80 (b) $\star 100$

(c) 120 (d) 160

(e) None of the above

**18.** Five times Hannah's money added to Darby's money is more than \$51.00. Three times Hannah's money minus Darby's money is \$21.00. If $h$ represents Hannah's money (in dollars) and $d$ represents Darby's money (in dollars), then:

(a) $\star h > 9$, $d > 6$

(b) $h > 9$, $d < 6$

(c) $h > 9$, $d = 6$

(d) $h > 9$, but we can put no bounds on $d$.

(e) None of the above

**19.** Find $a+b$ where $(a, b)$ is a point in the first quadrant which is on the circle $x^2 + y^2 = 4$ and which satisfies $|f(a, b) - f(0, 2)| = \ln(2)$ where $f(x, y) = \ln(\frac{x+2}{y})$.

(a) $\frac{11}{5}$ (b) $\frac{12}{5}$

(c) $\frac{13}{5}$ (d) $\star\frac{14}{5}$

(e) None of the above

**20.** Assume that the following statements are true:

1. All children are human.
2. All students are human.
3. Some students eat.

Given the following four statements:

a. All children are students.
b. Some humans eat.
c. No children eat.
d. Some humans who eat are not children.

Those which are logical consequences of 1., 2. and 3. above are:

(a) $\star$b (b) d

(c) b,c (d) b,d

(e) None of the above

**21.** If we write $|x^2 - 9| < N$ for all $x$ such that $|x - 3| < 0.02$, the smallest value we can use for $N$ is:

(a) .02 (b) .0612
(c) .1122 (d) $\star$.1204
(e) None of the above

**22.** If three of the roots of $x^4 + ax^2 + bx + c = 0$ are 1, 2, and 3, then the value of $a + c$ is:

(a) 35 (b) 24
(c) $-12$ (d) $\star{-61}$
(e) None of the above

**23.** Given that $f((x-1)^{-1}) = x^{-1}$, then $f(x)$ is:

(a) $(x-1)^{-1}$ (b) $\star\frac{x}{x+1}$
(c) $\frac{x+1}{x}$ (d) $\frac{1}{x} - x$
(e) None of the above

**24.** How many different signals, each consisting of six flags hung in a vertical line, can be formed from four identical red flags and two identical blue flags?

(a) 12 (b) $\star$15
(c) 18 (d) 21
(e) None of the above

**25.** Suppose that $f(n) = \log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdots \log_{n-1} n$. Then $\sum_{k=2}^{9} f(2^k) =$

(a) 41 (b) 42
(c) 43 (d) $\star$44
(e) None of the above

**26.** Let $f(x) = |x - a| + |x - 10| + |x - a - 10|$, where $a$ is some number satisfying $0 < a < 10$. What is the minimum value taken by $f$?

(a) $\star$10 (b) $a$
(c) $10 - a$ (d) $10 + a$
(e) None of the above

**27.** The largest root of $f(t) = 2t^3 - 3t^2 - 6t - 2$ has the form $1 + \frac{a}{2}$. Find $a$, given that the polynomial has at least one rational root.

(a) $\sqrt{3}$ (b) $\star 2\sqrt{3}$
(c) $\sqrt{2}$ (d) $2\sqrt{2}$
(e) None of the above

**28.** Find the number of ways that a class of 10 students can be partitioned into two teams of size two and two teams of size three.

(a) $\star$6300 (b) 12900
(c) 25200 (d) 50400
(e) None of the above

**29.** The base of a triangle has length 80, and one of the base angles is 60°. The sum of the lengths of the other two sides is 90. The shortest side is:

(a) 45 (b) 40
(c) 36 (d) $\star$17
(e) None of the above

**30.** In triangle $ABC$, $D$ is between $C$ and $B$, $AC = CD$ and $m(\angle CAB) - m(\angle ABC) = 30°$. Then $m(\angle BAD) =$

(a) 30° (b) 20°
(c) 22.5° (d) 10°
(e) None of the above

**31.** There are 12 points including $A$ in a plane. No three of the points are collinear. How many triangles are there determined by these points (as vertices) which have $A$ as a vertex?

(a) 40 (b) 50
(c) $\star$55 (d) 220
(e) None of the above

**32.** The largest real root of $f(t) = t^4 - 3t^3 + 6t^2 + 25t - 39$ has the form $\frac{-1+a}{2}$ where $a > 0$. Given that $2 - 3i$ is a root of this polynomial, find $a$.

(a) $\sqrt{10}$ (b) $\sqrt{11}$
(c) $\sqrt{12}$ (d) $\star\sqrt{13}$
(e) None of the above

**33.** What is the largest 2-digit prime factor of $\binom{200}{100}$?

(a) 59 (b) $\star$61

(c) 65 (d) 67

(e) None of the above

**34.** Suppose that $f(x) = x^4 + ax^3 + bx^2 + cx + d$, and that $f(1) = f(2) = f(3) = f(4)$. Then $b =$

(a) $\star$35 (b) 36

(c) 37 (d) 38

(e) None of the above

**35.** Find the sum of the digits of the only even 3−digit number whose digits are consecutive integers, possibly rearranged, such that the sum of the first and third digits equals three times the middle digit.

(a) $\star$12 (b) 9

(c) 15 (d) 21

(e) None of the above

**36.** The slope of the line which makes an angle of 120° with the line $-3y - x = 4$ has the form $-\frac{a+b\sqrt{3}}{3}$. What is $a + b$?

(a) $\star$11 (b) 12

(c) 13 (d) 14

(e) None of the above

**37.** If $\tan(x + y) = 33$ and $\tan(x) = 3$, find $\tan(y)$.

(a) 30 (b) 3

(c) $\star$.3 (d) .33

(e) None of the above

**38.** Find the smallest positive value of $x$ (in degrees) for which $\tan(x) + 3\cot(x) = 4$.

(a) 25 (b) 40

(c) 125 (d) 225

(e) None of the above

**39.** $P$ is the midpoint of $\overline{AB}$, which has length 2 and is the shortest side of a $30 - 60 - 90$ triangle $ABC$. What is the smallest possible perimeter of a triangle with one vertex at $P$ and the other two vertices on $\overline{AC}$ and $\overline{BC}$ respectively?

(a) $\star\sqrt{13}$ (b) $\sqrt{14}$

(c) $\sqrt{15}$ (d) 4

(e) None of the above

**40.** If $a \geq 1$, then the sum of the real solutions of

$$\sqrt{a - \sqrt{a + x}} = x$$

is equal to:

(a) $\sqrt{a} - 1$ (b) $\frac{\sqrt{a}-1}{2}$

(c) $\sqrt{a - 1}$ (d) $\frac{\sqrt{a-1}}{2}$

(e) None of the above

## Bonus Questions

**41.** How many regular polygons have the same number of diagonals as sides? Find, with proof, all of them.

**42.** Given $\triangle ABC$ with $AB < AC$, if $D$ is a point on $\overline{BC}$ between $B$ and $C$, prove that $AD < AC$.