1. What is the smallest positive solution x of the equation $5\cos x - 2\cos^2 x = 2$?

(a) $\star\frac{\pi}{3}$ (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$
(e) None of the above

2. A function satisfies $f(0, n) = n + 1$ and $f(m+1, n) = f(m, f(m, n))$. What is $f(4, 100)$?

(a) $\star 116$ (b) 118
(c) 120 (d) 122
(e) None of the above

3. Let A be the area of the region bounded by the x axis, the line $x = 8$, and the graph of the function f where f is defined by the rule $f(x) = x$ when $0 \leq x \leq 5$ and $f(x) = 2x - 5$ when $5 \leq x \leq 8$. Then A is:

(a) 21.5 (b) 36.4
(c) $\star 36.5$ (d) 44
(e) None of the above

4. Suppose that the equation of the circle having $(-3, 5)$ and $(5, -1)$ as end points of a diameter is $(x-a)^2+(y-b)^2 = r^2$. Then $a + b + r$ is

(a) $\star 8$ (b) 9
(c) 10 (d) 11
(e) None of the above

5. The product of the solutions to the quadratic equation $ax^2 + bx + c = 0$ is 6. The product of the solutions of $bx^2 + cx + a = 0$ is 8. What is the product of the solutions of $cx^2 + ax + b = 0$?

(a) $\frac{1}{42}$ (b) $\frac{1}{46}$
(c) $\star\frac{1}{48}$ (d) $\frac{1}{50}$
(e) None of the above

6. The sum of the first n terms of the sequence

$$1, (1+2), (1+2+2^2), (1+2+2^2+2^3), \ldots, (1+2+\ldots+2^{n-1})$$

in terms of n is:

(a) 2^n (b) $2^n - n$
(c) $2^{n+1} - n$ (d) $\star 2^{n+1} - n - 2$
(e) None of the above

7. The roots of $64x^3 - 144x^2 + 92x - 15 = 0$ are in arithmetic progression. The difference between the largest and smallest roots is:

(a) 2 (b) $\star 1$
(c) $\frac{1}{2}$ (d) $\frac{3}{8}$
(e) None of the above

8. A positive number x is mistakenly divided by 6 instead of being multiplied by 6. Based on the correct answer, the error thus committed, to the nearest percent, is:

(a) 100 (b) $\star 97$
(c) 83 (d) 17
(e) None of the above

9. Let $A = 4^{(3^2)}$, $B = 3^{(4^2)}$, and $C = 2^{(3^4)}$. List A, B, and C in increasing order.

(a) $\star A\ B\ C$ (b) $A\ C\ B$
(c) $B\ A\ C$ (d) $B\ C\ A$
(e) None of the above

10. If the graphs of $2y + x + 3 = 0$ and $3y + ax + 2 = 0$ are to meet at right angles, the value of a is:

(a) $\frac{-2}{3}$ (b) $\frac{-3}{2}$
(c) 6 (d) $\star -6$
(e) None of the above

11. Let z_1 be the complex number $1-i$, and let $z_{n+1} = 1 - iz_n$ for $n \geq 1$. What is z_{19}?

(a) $\star 0$ (b) $1 - i$
(c) $-i$ (d) $1 + i$
(e) None of the above

12. If $x > y > 0$ and $2\log(x - y) = \log x + \log y$, then $x/y =$

(a) $3 + \sqrt{5}$ (b) $\star\frac{1}{2}(3 + \sqrt{5})$
(c) $3 - \sqrt{5}$ (d) $\frac{1}{2}(3 - \sqrt{5})$
(e) None of the above

13. For all pairs of angles (A, B), measured in degrees, such that $\sin A + \sin B = \sqrt{2}$ and $\cos A + \cos B = \sqrt{\sqrt{2}}$, both hold simultaneously, what is the smallest possible value of $|A - B|$ in degrees?

(a) 15 (b) 30

(c) $\star$45 (d) 90

(e) None of the above

14. If we write $|x^2 - 4| < N$ for all x such that $|x - 2| < 0.01$, the smallest value we can use for N is:

(a) .0301 (b) .0349

(c) .0399 (d) $\star$.0401

(e) None of the above

15. A circle is inscribed in an equilateral triangle, and a square is inscribed in the circle. The ratio of the area of the triangle to the area of the square is:

(a) $\sqrt{3}$:1 (b) $\sqrt{3}$:$\sqrt{2}$

(c) $\star 3\sqrt{3}$:2 (d) 3:$\sqrt{2}$

(e) None of the above

16. The number of solutions of $2^{2x} - 3^{2y} = 55$, in which x and y are integers, is:

(a) 0 (b) $\star$1

(c) 2 (d) 3

(e) None of the above

17. What is the value of

$$\sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{\cdots}}}}$$

(a) 1 (b) 2

(c) $\star$3 (d) 4

(e) None of the above

18. The angle of elevation to the top of a building is 45°. If you move 20 feet farther away the angle of elevation becomes 30°. What is the height of the building (in feet)?

(a) 30 (b) 25

(c) $10(\sqrt{2} + 1)$ (d) $\star 10(\sqrt{3} + 1)$

(e) None of the above

19. Out of all ways of arranging 5 points inside an equilateral triangle of side length 2, what is the smallest number of pairs of these points which could be at distance ≤ 1 from one another?

(a) 0 (b) $\star$1

(c) 2 (d) 3

(e) None of the above

20. The smallest positive integer x for which $1260x = N^3$ where N is an integer, is:

(a) 1050 (b) 1260

(c) $\star$7350 (d) 44100

(e) None of the above

21. The area of a circle inscribed in an equilateral triangle is 48π. What is the perimeter of the triangle?

(a) 64 (b) 68

(c) 70 (d) $\star$72

(e) None of the above

22. The tens digit of 2^{100} is:

(a) 4 (b) 5

(c) 6 (d) $\star$7

(e) None of the above

23. Which of the following is equivalent to $\sqrt{7 - 4\sqrt{3}}$?

(a) $\sqrt{7} - 2 \cdot \sqrt{3}$ (b) $\sqrt{7} - \sqrt{3}$

(c) $7 - \sqrt{3}$ (d) $\star 2 - \sqrt{3}$

(e) None of the above

24. A three – digit number has, from left to right, the digits h, t, and u with $h > u$. When the number with the digits reversed is subtracted from the original number, the units' digit in the difference is 4. The next two digits, from right to left, are:

(a) 5 and 9

(b) $\star$9 and 5

(c) impossible to tell

(d) 5 and 4

(e) None of the above

25. The current in a river is flowing steadily at 3 miles per hour. A motor boat which travels at a constant rate in still water goes downstream 4 miles and then returns to its starting point. The trip takes one hour, excluding the time spent in turning the boat around. The ratio of the downstream to the upstream rate is:

(a) 4:3 (b) 3:2
(c) 5:3 (d) ⋆2:1
(e) None of the above

26. For $x^2 + 2x + 5$ to be a factor of $x^4 + px^2 + q$, the values of p and q must be, respectively:

(a) -2, 5 (b) 5, 25
(c) 10, 20 (d) ⋆6, 25
(e) None of the above

27. For how many positive real values of x does $\log_4 x = 2 \sin x$?

(a) 4 (b) ⋆5
(c) 6 (d) 7
(e) None of the above

28. Working in base 100, we evaluate 11^{20}. Note that both the 11 and 20 here are base 100 numbers. What is the third digit from the right end, i.e., the coefficient of 100^2? Express this digit as a base 10 number.

(a) 1 (b) ⋆2
(c) 3 (d) 4
(e) None of the above

29. For a given arithmetic series the sum of the first 50 terms is 200, and the sum of the next 50 terms is 2700. the first term of the series is:

(a) -12221 (b) -21.5
(c) ⋆-20.5 (d) 3
(e) None of the above

30. A person starting with 64 cents and making 6 bets, wins three times and loses three times, the wins and losses occurring in random order. The chance for a win is equal to the chance for a loss. If each wager is for half the money remaining at the time of the bet, then the final result is:

(a) a loss of 27 cents
(b) a gain of 27 cents
(c) ⋆a loss of 37 cents
(d) neither a gain nor a loss
(e) None of the above

31. How many points P in the region bounded by a circle of radius 1 have the property that the sum of the squares of the distances from P to the endpoints of a given diameter is $2\sqrt{2}$?

(a) 0 (b) 5
(c) 10 (d) ⋆Infinitely many.
(e) None of the above

32. How many solutions in positive integers are there for the equation $2x + 3y = 763$?

(a) 255 (b) 254
(c) 128 (d) ⋆127
(e) None of the above

33. Let S be the solution set of the inequality

$$|x^2 - 8x| \geq |x^2 - 8x + 2|.$$

What is the smallest value of r for which S is also the solution set of the inequality $|x^2 - 8x + c| \leq r$ for some real number c?

(a) 13/2 (b) ⋆15/2
(c) 8 (d) 17/2
(e) None of the above

34. The set of all x for which $\frac{3}{x} > \frac{10}{x^2+1}$ consists of the union of a finite and an infinite interval. The length of the finite interval is

(a) 3 (b) $2\frac{1}{3}$
(c) ⋆$\frac{1}{3}$ (d) $2\frac{2}{3}$
(e) None of the above

35. Let

$$f(n) = \frac{5+3\sqrt{5}}{10}\left(\frac{1+\sqrt{5}}{2}\right)^n + \frac{5-3\sqrt{5}}{10}\left(\frac{1-\sqrt{5}}{2}\right)^n.$$

Then $f(n+1) - f(n-1)$, expressed in terms of $f(n)$, is:

(a) $\frac{1}{2}f(n)$ (b) $\star f(n)$

(c) $2f(n)+1$ (d) $f^2(n)$

(e) None of the above

36. If $\tan x = \frac{2ab}{a^2-b^2}$, where $a > b > 0$ and $0° < x < 90°$, then $\sin x$ is equal to:

(a) $\frac{a}{b}$ (b) $\frac{\sqrt{a^2-b^2}}{2a}$

(c) $\frac{b}{a}$ (d) $\star\frac{2ab}{a^2+b^2}$

(e) None of the above

37. Triangle ABC is isosceles with base AC. Points P and Q are respectively in CB and AB and such that $AC = AP = PQ = QB$. The number of degrees in angle B is:

(a) $\star 25\frac{5}{7}$

(b) 30

(c) 40

(d) Not enough information to tell.

(e) None of the above

38. Evaluate $\sum_{k=1}^{19} \frac{(-2)^k}{k!(19-k)!}$ as a simple fraction.

(a) .00000002 (b) $\frac{1}{19!}$

(c) $\star\frac{-2}{19!}$ (d) 1

(e) None of the above

39. Given the sets of consecutive integers $\{1\}$, $\{2,3\}$, $\{4,5,6\}$, etc. where each set contains one more element than the preceding one, and where the first element of each set is one more than the last element of the preceding set. Let S_n be the sum of the elements of the nth set. Then S_{21} is:

(a) 1113 (b) $\star$4641

(c) 5082 (d) 53361

(e) None of the above

40. If the line $y = mx + 1$ intersects the ellipse $x^2 + 4y^2 = 1$ exactly once, then the value of m^2 is:

(a) 1/2 (b) 2/3

(c) $\star$3/4 (d) 4/5

(e) None of the above

Bonus Questions

41. A set of consecutive positive integers beginning with 1 is written on a blackboard. One number is erased. The average of the remaining numbers is $35\frac{7}{17}$. Find, with proof, the number that was erased.

Answer: 7.

42. Find, with proof, all sets of two or more consecutive positive integers whose sum is 100.

Answer: $\{18,\ldots,22\}$ and $\{9,\ldots,16\}$