1. What is the center of the circle which circumscribes the triangle with vertices $(2,0)$, $(0,4)$, and $(4,6)$?

(a) (3,3) (b) (2,2)

(c) (1,1) (d) (3,2)

(e) None of the above

2. A long, level highway bridge passes over a railroad track which is 100 feet below it and at right angles to it. If an automobile traveling 66 feet per second is directly above a train going 88 feet per second, how far apart are they 10 seconds later?

(a) $\sqrt{100^2+880^2}$ ft. (b) $100\sqrt{122}$ ft.

(c) 1000 ft. (d) 1100 ft.

(e) None of the above

3. The solution set of $1+x+x^2+x^3+\ldots+x^{1997}\leq 0$ is

(a) $\{x : x\leq -2\}$ (b) $\{x : x\leq -1\}$

(c) $\{x : -1\leq x\leq 0\}$ (d) $\{x : x\leq 0\}$

(e) None of the above

4. Consider the set of points with the property that they are twice as far from $(3,4)$ as they are from $(1,1)$. These form a circle with what radius?

(a) $2\sqrt{13}$ (b) $\sqrt{26}$.

(c) $\sqrt{52}/3$ (d) $\sqrt{26}/2$

(e) None of the above

5. Consider a set of non-intersecting circles of radius r with centers at the vertices of a convex n – sided polygon having sides of lengths d_1, $d_2, \ldots, d_n$. (with $r<\min\{d_1, d_2, \ldots d_n\}/2$.) How long is the taut belt that fits around these circles?

(a) $\pi r/2+\sum_{i=1}^{n} d_i$

(b) $\pi r+\sum_{i=1}^{n} d_i$

(c) $2\pi r+\sum_{i=1}^{n} d_i$

(d) $\pi r/n+\sum_{i=1}^{n} d_i$

(e) None of the above

6. Express the perpendicular distance between the parallel lines $y=mx+b$ and $y=mx+B$ in terms of m, b, and B.

(a) $|B-b|$ (b) $\frac{|B-b|}{m}$

(c) $\frac{|b-B|}{\sqrt{m^2+1}}$ (d) $\frac{|b-B|}{\sqrt{m^2-1}}$

(e) None of the above

7. An isosceles triangle AOB with $|AB|=|BO|$ is topped by a semicircle, as shown in Figure 1. Let D be the area of triangle AOB and E the area of the semicircle. Find a formula for E/D in terms of t. (the angle shown).

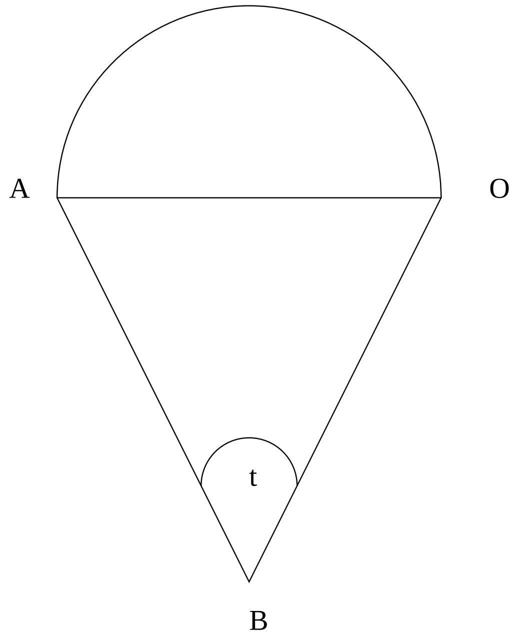

Fig. 1

(a) $\frac{\pi}{2}\tan(t/2)$ (b) $\pi\tan(t)$

(c) $\pi\tan(t/2)$ (d) $\frac{\pi}{2}\tan(t)$

(e) None of the above

8. Find the sum of the two largest real solutions of

$$x^5+8x^4+6x^3-42x^2-19x-2=0.$$

(a) -1 (b) $-1+2\sqrt{2}$

(c) $\frac{-1+2\sqrt{2}}{2}$ (d) $2\sqrt{2}$

(e) None of the above

9. A pint mug A is filled with milk. A fraction k of a pint is put into a different, previously empty pint mug B. Mug B is then filled with water and the contents are mixed, and then mug A is topped off with the solution from mug B. This is done in such a way as to minimize the amount of milk in mug A. How much milk is in mug A?

(a) 1/2 of a pint. (b) 2/3 of a pint.

(c) 3/4 of a pint. (d) 4/5 of a pint.

(e) None of the above

10. Figure 2 shows a quarter circle of radius 2 in which is inscribed a semicircle of radius 1. A smaller semicircle is then inscribed as shown. The radius of this smaller semicircle is:

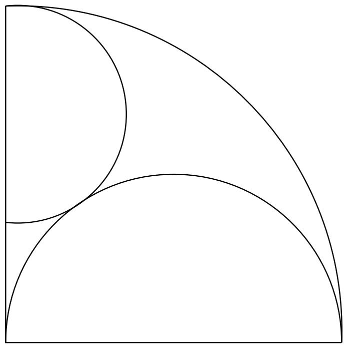

Fig. 2

(a) 4/3 (b) 1/2

(c) 4/5 (d) 2/3

(e) None of the above

11. A poster is taped flush on a wall that is leaning toward the observer at an angle of 15 degrees off vertical. If from a point on the floor 6 feet from the base of the wall the line of sight to the top of the poster makes an angle of 45 degrees with the floor while the line of sight to the bottom of the poster makes an angle of 30 degrees with the floor, then in feet from top to bottom, find the length of the poster.

(a) $3\sqrt{2}-\sqrt{6}$ (b) $2\sqrt{6}/3$

(c) $6-2\sqrt{3}$ (d) 2

(e) None of the above

12. Each of the five small squares in Figure 3 below is to be assigned a number from the list $\{1,2,3\}$. The number of ways that this can be done so that no two squares with a common edge contain the same number is:

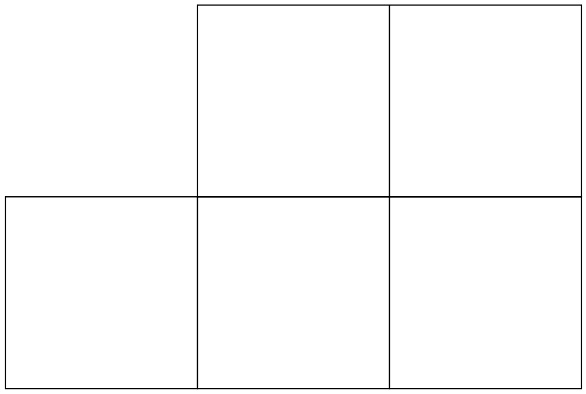

Fig. 3

(a) 24 (b) 36

(c) 48 (d) 60

(e) None of the above

13. A five-digit zip code is said to be detour-prone if it looks like a valid and different zip code when read upside down. For instance 68901 and 88111 are detour-prone while 32145 and 10801 are not. How many of the 10^5 possible zip codes are detour-prone?

(a) 3050 (b) 3100

(c) 3150 (d) 3200

(e) None of the above

14. Two perpendicular chords intersect in a circle. The segments of one chord are 3 and 4; the segments of the other are 6 and 2. The diameter of the circle is...

(a) $\sqrt{89}$ (b) $\sqrt{56}$

(c) $\sqrt{61}$ (d) $\sqrt{65}$

(e) None of the above

15. Solve $16^x + 4^x = y$ for x.

(a) $x = 4$.

(b) $x = \frac{1}{2}\frac{\ln y}{\ln 4}$.

(c) $x = \frac{\ln y + .25}{\ln 16}$.

(d) $x = \frac{\ln y + 4}{\ln 16}$.

(e) None of the above

16. What is the thousands' digit of 101^{29}?

(a) 1 (b) 3
(c) 7 (d) 9
(e) None of the above

17. A check is written for x dollars and y cents where x and y are both two digit numbers. In error it is cashed for y dollars and x cents, the incorrect amount exceeding the correct amount by \$17.82. Then:

(a) x cannot exceed 70
(b) y can equal $2x$
(c) the amount of the check cannot be a multiple of 5
(d) y can equal $3x$
(e) None of the above

18. A chemist has m ounces of salt water that is $m\%$ salt. How many ounces of salt must he add to make a solution that is $2m\%$ salt?

(a) $\frac{m}{100+m}$ (b) $\frac{2m}{100-2m}$
(c) $\frac{m^2}{100+2m}$ (d) $\frac{m^2}{100-2m}$
(e) None of the above

19. Let a be the total number of cents I give you if I give you k cents on the kth day for 100 days. Let b be the total number of cents I give you if I give you 1 cent on the first day, and twice as many on the $(k+1)$st day as I gave you on the kth day for 20 days. Let c be the number of hours which transpire in the course of two centuries. Which of these is the biggest?

(a) a (b) b
(c) c (d) 2,000,000
(e) None of the above

20. A bag has 8 pennies and a quarter in it. 4 coins are picked at random in such a way that all are equally likely to be picked, and these are put in an urn which contains 12 pennies. 3 coins are then picked from the urn and put back in the bag. What is the probability that the quarter ends up in the bag?

(a) 20/36 (b) 21/36
(c) 22/36 (d) 23/36
(e) None of the above

21. John, who is 6 feet tall, notices that his shadow is 10 feet long. He moves into the shadow of the school flagpole until the shadow of the top of his head coincides with the shadow of the top of the flagpole (his shadow remains 10 feet long after he moves.) He then measures the distance from where he stands to the base of the flagpole and finds it to be 90 feet. The height of the flagpole, in feet, is:

(a) 50 (b) 30
(c) 110 (d) 60
(e) None of the above

22. Six circles of radius r feet are placed as shown in Figure 4. Let A be the area bounded by but not included in the circles (in square feet), and let B be the height of the stack (in feet). Find $A + Br$.

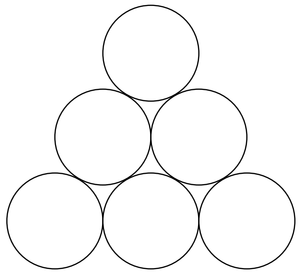

Fig. 4

(a) $2\sqrt{12} - 2\pi r^2$ (b) $2r + \sqrt{12}r$
(c) $2r + \sqrt{12}r + 2\sqrt{12}r^2$ (d) $r^2(2 + 3\sqrt{12} - 2\pi)$
(e) None of the above

23. Let k be the smallest positive integer with the property that for all n such that $2 \le n \le 10$, when divided by n it leaves a remainder of $n-1$. Find the sum of the digits of k.

(a) 12 (b) 13
(c) 15 (d) 17
(e) None of the above

24. Four positive integers a, b, c, d are given. There are exactly 4 distinct ways to choose 3 of a, b, c, d. The mean of each of the four possible triples is added to the 4th integer. The four sums 29, 23, 21, 17 are obtained. One of the original integers is:

(a) 19 (b) 21
(c) 23 (d) 29
(e) None of the above

25. Consider a recursive function C defined by

$$C(n,0) = n+1$$
$$C(0,i) = C(1,i-1) \text{ for } i > 0$$
$$C(n,i) = C(C(n-1,i),i-1) \text{ for } n > 0 \text{ and } i > 0.$$

Compute $C(1,2)$ from this definition.

(a) 2 (b) 3

(c) 4 (d) 5

(e) None of the above

26. If $(\log_3 x)(\log_x 2x)(\log_{2x} y) = \log_x x^2$ then y is

(a) 9/2 (b) 9

(c) 18 (d) 27

(e) None of the above

27. Given sets M and N, define $M \nabla N$ to be

$$(M-N) \cup (N-M).$$

Let S be any finite set and T be the set of all subsets of S. Let $\clubsuit$ be the statement that ∇ is associative on T, let $\diamondsuit$ be the statement that ∇ is commutative on T, let $\heartsuit$ be the statement that there is an element in T which acts like an identity element with respect to ∇,and let $\spadesuit$ be the statement that every element in T is its own inverse with respect to ∇. Then:

(a) Only $\clubsuit$ is true

(b) Only $\clubsuit, \diamondsuit, \heartsuit$ are true

(c) Only $\clubsuit$ and $\diamondsuit$ are true

(d) All of $\clubsuit$, $\diamondsuit$, $\heartsuit$, and $\spadesuit$ are true.

(e) None of the above

28. Let $D = a^2+b^2+c^2$ where a and b are consecutive integers and $c = ab$. Then $\sqrt{D}$ is:

(a) always an even integer

(b) sometimes an odd integer, sometimes not

(c) always an odd integer

(d) sometimes a rational number, sometimes not

(e) None of the above

29. $\log_4 \sqrt[5]{64 \cdot \sqrt[4]{128 \cdot \sqrt{32 \cdot 16^{(-.75)}}}} =$

(a) 2/3 (b) 3.0303

(c) 1/5 (d) 3.030303

(e) None of the above

30. If P is a point interior to rectangle $ABCD$ and such that $|PA| = 3$, $|PD| = 4$, $|PC| = 5$, then $|PB| =$?.

(a) $2\sqrt{3}$ (b) $3\sqrt{2}$

(c) $3\sqrt{3}$ (d) $4\sqrt{2}$

(e) None of the above

31. A watch loses 2.5 minutes per day. It is set right at 1 p.m. on March 15th. Let n be the positive correction, in minutes, to be added to the time shown by the watch at a given time. When the watch shows 9:00 a.m. on March 21, n equals what?

(a) $14\frac{14}{23}$ (b) $14\frac{1}{14}$

(c) $13\frac{101}{115}$ (d) $13\frac{83}{115}$

(e) None of the above

32. If $\sin(a) = 2/3$ and $\cos(b) = 3/4$ and a is in quadrant II and b is in quadrant IV, then $\sin(a-b)$ is

(a) $\frac{6+\sqrt{35}}{12}$ (b) $\frac{-3\sqrt{5}+2\sqrt{7}}{12}$

(c) $\frac{6-\sqrt{35}}{12}$ (d) $\frac{-3\sqrt{5}-2\sqrt{7}}{12}$

(e) None of the above

33. The solution set of the inequality $|2x-1| - |x+3| < 4$ can be written as the union of (possibly infinite) intervals. The sum of the lengths of these intervals is:

(a) 3.5 (b) 6

(c) 7.5 (d) 10

(e) None of the above

34. A game of poker is played with a deck of 52 cards, consisting of 4 suits of 13 card each. A "hand" consists of 5 cards selected from the deck at random, and a "flush" is a hand which contains all 5 cards of the same suit. How many hands are flushes?

(a) 52 (b) 13!

(c) 1287 (d) 5148

(e) None of the above

35. Three fair dice, one white, one red and one black, are tossed and the number of spots on the top faces are noted. Which of the following have probability 1/4 :

(a) The red die is a six or the black die is even.

(b) All three numbers are even.

(c) The red die is even and the black die is 4 or more.

(d) Exactly two of the three numbers are equal.

(e) None of the above

36. How far is it from the intersection of the lines

$$2x + 3y = 3$$
$$3x - 5y = 14$$

to the center of the circle $x^2 + y^2 + 4y + 1 = 20$?

(a) $\sqrt{10}$ (b) 10

(c) $3\sqrt{2}$ (d) $\sqrt{20}$

(e) None of the above

37. The four digit number $2pqr$ (i.e., the one's digit is $r, \ldots$) is multiplied by 4 and the result is the 4 digit number $rqp2$. It follows that $p + q =$?

(a) 8 (b) 7

(c) 6 (d) 5

(e) None of the above

38. The numeral 47 in base a represents the same number as 74 in base b. Assuming that both bases are integers greater than one, the least possible value for $a + b$ is:

(a) 2 (b) 13

(c) 20 (d) 24

(e) None of the above

39. The number $2^{48} - 1$ is divisible by two numbers between 60 and 70. Their sum is:

(a) 124 (b) 128

(c) 132 (d) 136

(e) None of the above

40. Tom always tells the truth, Dick sometimes tells the truth, and Harry never tells the truth. On the way to a masquerade party, the one dressed as Bob Dole says "Tom is Bill Clinton." The one dressed as Bill Clinton says "I'm Dick." The one dressed as Ross Perot says "Harry is Bill Clinton." The man dressed as Bill Clinton is:

(a) Tom (b) Dick

(c) Harry (d) Either Tom or Dick

(e) None of the above

Bonus Questions

41. Consider the set

$$P = \{(x, y) \mid x \text{ and } y \text{ are real numbers and } y > 0\}.$$

For real numbers c and r, let $L_{c,r}$ be the set of points of the form

$$\{(x, y) \in P \mid (x - c)^2 + y^2 = r^2\}.$$

Define L_a to be the set of points of the form

$$\{(a, y) \in P \mid y \text{ is a real number}\}.$$

Define a blurb to be any of the sets $L_{c,r}$ or L_a. Show that every pair of points in P determines a unique blurb.

42. Using the notation of the previous problem, classify all blurbs which go through the point (0,1) and don't intersect L_6.