

1. The price of an item is marked up by 10% and then by another 20%. What is the overall markup (in %)?
 - (a) 30
 - (b) 31
 - (c) $\star 32$
 - (d) 33
 - (e) None of the above
2. The number of digits in the number $N = 2^{12} \cdot 5^8$ is:
 - (a) 9
 - (b) $\star 10$
 - (c) 11
 - (d) 12
 - (e) None of the above
3. Mark can clean out the barn in 40 minutes. Suzan can clean out the same barn in 32 minutes. How many minutes will it take the two of them working together?
 - (a) $\star 160/9$
 - (b) 16
 - (c) 160/11
 - (d) 160/12
 - (e) None of the above
4. Suppose that the equation of the circle having $(-3, 5)$ and $(5, -1)$ as end points of a diameter is $(x-a)^2 + (y-b)^2 = r^2$. Then $a + b + r$ is
 - (a) $\star 8$
 - (b) 9
 - (c) 10
 - (d) 11
 - (e) None of the above
5. An automobile travels $\frac{a}{6}$ feet in r seconds. If this rate is maintained for 3 minutes, how many yards does it travel in the 3 minutes?
 - (a) $\frac{30r}{a}$
 - (b) $\frac{30a}{r}$
 - (c) $\frac{10r}{a}$
 - (d) $\star \frac{10a}{r}$
 - (e) None of the above
6. Suppose that $f(x) = 1 - \frac{1}{(1-x)}$. Find $f(f(f(\dots f(3)\dots)))$, where there are 1998 f 's in this composition.
 - (a) $\star 3$
 - (b) $3/2$
 - (c) $2/3$
 - (d) 1
 - (e) None of the above
7. Let p units be the increase in the circumference of a circle resulting from an increase of π units in the diameter. Then p equals:
 - (a) $\frac{1}{\pi}$
 - (b) π
 - (c) $\frac{\pi^2}{2}$
 - (d) $\star \pi^2$
 - (e) None of the above
8. A positive number x is mistakenly divided by 6 instead of being multiplied by 6. Based on the correct answer, the error thus committed, to the nearest percent, is:
 - (a) 100
 - (b) $\star 97$
 - (c) 83
 - (d) 17
 - (e) None of the above
9. Let $A = 4^{(3^2)}$, $B = 3^{(4^2)}$, and $C = 2^{(3^4)}$. List A, B, and C in increasing order.
 - (a) $\star A B C$
 - (b) $A C B$
 - (c) $B A C$
 - (d) $B C A$
 - (e) None of the above
10. If the graphs of $2y + x + 3 = 0$ and $3y + ax + 2 = 0$ are to meet at right angles, the value of a is:
 - (a) $\frac{-2}{3}$
 - (b) $\frac{-3}{2}$
 - (c) 6
 - (d) $\star -6$
 - (e) None of the above
11. 6 women and 12 men are on a committee from which a subcommittee of size 6 is to be chosen. If at least 2 members of the subcommittee must be men, and if there must be at least as many women as men on the subcommittee, how many ways can the subcommittee be chosen?
 - (a) 5380
 - (b) $\star 5390$
 - (c) 5400
 - (d) 5420
 - (e) None of the above
12. The sum of three numbers is 98. The ratio of the first to the second is $\frac{2}{3}$, and the ratio of the second to the third is $\frac{5}{8}$. The second number is:
 - (a) 15
 - (b) 20
 - (c) $\star 30$
 - (d) 32
 - (e) None of the above

- 13.** The number of ounces of water one needs to add to 9 ounces of shaving lotion containing 50% alcohol to create a lotion containing 30% alcohol is:
- (a) 3 (b) 4
(c) 5 (d) $\star 6$
(e) None of the above
- 14.** A runner runs around a circular track in the same amount of time that a slower runner takes to run around a smaller concentric circular track. The faster runner is twice as fast as the slower runner, and is running on a track of circumference 100π yards. How many yards apart are the two tracks?
- (a) 20 (b) $\star 25$
(c) 30 (d) 35
(e) None of the above
- 15.** A circle is inscribed in an equilateral triangle, and a square is inscribed in the circle. The ratio of the area of the triangle to the area of the square is:
- (a) $\sqrt{3}:1$ (b) $\sqrt{3}:\sqrt{2}$
(c) $\star 3\sqrt{3}:2$ (d) $3:\sqrt{2}$
(e) None of the above
- 16.** Suppose $f(x) = x^4 - x^3 + ax + b$, $f(1) = 4$, and $f(2) = 6$. What is a ?
- (a) 6 (b) 2
(c) -2 (d) $\star -6$
(e) None of the above
- 17.** The solution set of the inequality
- $$\frac{(x-1)(x-10)^2}{x^3} < 0$$
- is
- (a) $-1 < x < 1$ (b) $\star 0 < x < 1$
(c) $-1 < x < 0$ (d) $0 < x < \sqrt{10}$
(e) None of the above
- 18.** The angle of elevation to the top of a building is 45° . If you move 20 feet farther away the angle of elevation becomes 30° . What is the height of the building (in feet)?
- (a) 30 (b) 25
(c) $10(\sqrt{2} + 1)$ (d) $\star 10(\sqrt{3} + 1)$
(e) None of the above
- 19.** The vertex of the parabola $y = x^2 - 8x + c$ will be a point on the x axis if the value of c is:
- (a) $\star 16$ (b) -16
(c) 4 (d) -4
(e) None of the above
- 20.** The smallest positive integer x for which $1260x = N^3$ where N is an integer, is:
- (a) 1050 (b) 1260
(c) $\star 7350$ (d) 44100
(e) None of the above
- 21.** Suppose that the lines which bisect the exterior angles at B and C of triangle ABC meet at D . Then the measure in degrees of angle BDC is:
- (a) $\frac{1}{2}(90 - m(\angle A))$ (b) $90 - m(\angle A)$
(c) $\star \frac{1}{2}(180 - m(\angle A))$ (d) $180 - m(\angle A)$
(e) None of the above
- 22.** The tens digit of 2^{100} is:
- (a) 4 (b) 5
(c) 6 (d) $\star 7$
(e) None of the above
- 23.** Which of the following is equivalent to $\sqrt{7 - 4\sqrt{3}}$?
- (a) $\sqrt{7} - 2 \cdot \sqrt{3}$ (b) $\sqrt{7} - \sqrt{3}$
(c) $7 - \sqrt{3}$ (d) $\star 2 - \sqrt{3}$
(e) None of the above
- 24.** The number of solutions to the following equation is:
- $$|||x - 1| - 2| - 3| = 1$$
- (a) 8 (b) 6
(c) 4 (d) 2
(e) None of the above

25. If $f(x) = \frac{2x}{3x+4}$ and $f(g(x)) = x$, then $g(x) = ?$

- (a) $\frac{3x+4}{2x}$ (b) $\frac{3x}{2x+4}$
 (c) $\frac{4x}{2-3x}$ (d) $\frac{2x+4}{4}$
 (e) None of the above

26. Consider the statements:

- I. $\sqrt{-4}\sqrt{-16} = \sqrt{(-4)(-16)}$
 II. $\sqrt{(-4)(-16)} = \sqrt{64}$
 III. $\sqrt{64} = 8$

Of these, the following are incorrect:

- (a) None (b) \star I. only
 (c) II. only (d) III. only
 (e) None of the above

27. The sum of two numbers is 10. Their product is 20. The sum of their reciprocals is:

- (a) $1/10$ (b) $\star 1/2$
 (c) 1 (d) 2
 (e) None of the above

28. What is the largest value that the function

$$f(x) = \frac{4}{x^2 + 6x + 12}$$

can assume?

- (a) $\frac{4}{12}$ (b) $\frac{4}{19}$
 (c) 4 (d) $\star \frac{4}{3}$
 (e) None of the above

29. Each of two angles of a triangle is 60° and the included side is 4 inches. The area of the triangle, in square inches, is:

- (a) $8\sqrt{3}$ (b) 8
 (c) $\star 4\sqrt{3}$ (d) 4
 (e) None of the above

30. If $f(x) = 4^x$, then $f(x+1) - f(x) =$

- (a) 4 (b) $f(x)$
 (c) $\star 3f(x)$ (d) $4f(x)$
 (e) None of the above

31. Amongst the set of all rectangles with perimeter 20 inches, the least value of any diagonal, in inches, is:

- (a) 5 (b) $\star\sqrt{50}$
 (c) 10 (d) $\sqrt{200}$
 (e) None of the above

32. If A is a set which has 32 subsets, B is a set which has 32 subsets, and $A \cup B$ has 256 subsets, then how many elements are in $A \cap B$?

- (a) 1 (b) $\star 2$
 (c) 3 (d) 4
 (e) None of the above

33. The number of solutions to

$$\{1, 2\} \subseteq X \subseteq \{1, 2, 3, 4, 5\}$$

where X is a set is:

- (a) 2 (b) 4
 (c) 6 (d) $\star 8$
 (e) None of the above

34. Given that there is a linear relationship between degrees Fahrenheit and degrees Celsius, and given that the freezing point of water is 0 degrees Celsius and 32 degrees Fahrenheit, and given that water boils at 100 degrees Celsius and 212 degrees Fahrenheit, find the Fahrenheit temperature that corresponds to 22 degrees Celsius (rounding to the nearest degree).

- (a) 69 (b) 70
 (c) 71 (d) $\star 72$
 (e) None of the above

35. How far from the origin is the line $3x + 4y = 100$?

- (a) $\star 20$ (b) 21
 (c) 22 (d) 23
 (e) None of the above

36. What is the sum of the real values of x satisfying the equality $|x + 3| = 2|x - 3|$?

- (a) 1 (b) $\star 10$
 (c) 12 (d) 13
 (e) None of the above

37. A series of 7 books are published at 9 – year intervals. When the 7th book was published, the sum of the publication years was 13601. In what year was the 5th book published?

- (a) 1907 (b) 1970
 (c) 1934 (d) 1943
 (e) None of the above

38. A jar contains 12 amoeba. The number of amoeba doubles every minute and it takes 30 minutes to fill the jar. How long (in minutes) does it take to fill $\frac{1}{4}$ the jar?

- (a) *28
 (b) 7.5
 (c) It can't be determined from the given information
 (d) $30 - \frac{120}{12}$
 (e) None of the above

39. A function is called even if $f(-x) = f(x)$ for all x and it is called odd if $f(-x) = -f(x)$ for all x . Which of the following are true statements:

- i) An even function times an odd function is even.
 ii) An even function plus an even function is even.
 iii) An odd function plus an odd function is even.

- (a) i) and ii) only
 (b) i) and iii) only
 (c) ii) and iii) only
 (d) All three are true.
 (e) None of the above

40. The graph of $\{(x, y) | 4x^2 + 4y^2 - 8x + 8y + 8 = 0\}$ in the real plane is:

- (a) A circle (b) A hyperbola
 (c) *A point (d) A parabola
 (e) None of the above

Answer: 7.

42. Find, with proof, all sets of two or more consecutive positive integers whose sum is 100.

Answer: $\{18, \dots, 22\}$ and $\{9, \dots, 16\}$

Bonus Questions

41. A set of consecutive positive integers beginning with 1 is written on a blackboard. One number is erased. The average of the remaining numbers is $35\frac{7}{17}$. Find, with proof, the number that was erased.