

1. Suppose that

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

and suppose that  $10 \leq R_1 \leq 20$ ,  $20 \leq R_2 \leq 30$ ,  $30 \leq R_3 \leq 40$ . Find the corresponding range for  $R$ .

- (a)  $60 \leq R \leq 90$                       (b)  $\frac{13}{120} \leq R \leq \frac{22}{120}$   
 (c)  $\frac{1}{90} \leq R \leq \frac{1}{60}$                       (d)  $\frac{120}{22} \leq R \leq \frac{120}{13}$   
 (e) None of the above

2. Suppose that  $x^2 + ax + y^2 + by + c = 0$  is that equation of a circle. What relationship must there be between  $a$ ,  $b$ , and  $c$ ?

- (a)  $c < a^2$                                       (b)  $c < \frac{a^2}{4}$   
 (c)  $c < \frac{a^2}{4} + \frac{b^2}{4}$                               (d)  $c < \frac{b^2}{4}$   
 (e) None of the above

3. Which of the following are always true?

- i)  $|a - b| \leq |a| + |b|$   
 ii)  $|a + b + c| \leq |a| + |b| + |c|$   
 iii)  $|a - b| \geq |a| - |b|$

- (a) i) only  
 (b) i) and ii) only  
 (c) All three of i), ii), and iii)  
 (d) i) and iii) only  
 (e) None of the above

4. Consider the set of points with the property that they are twice as far from  $(3, 4)$  as they are from  $(1, 1)$ . These form a circle with what radius?

- (a)  $2\sqrt{13}$                                       (b)  $\sqrt{26}$   
 (c)  $\sqrt{52}/3$                                       (d)  $\sqrt{26}/2$   
 (e) None of the above

5. Consider the triangle pictured in Figure 1. Given that angle ABC is a right angle and that segments AD and DC are each of length 3, then the length of line segment DB is

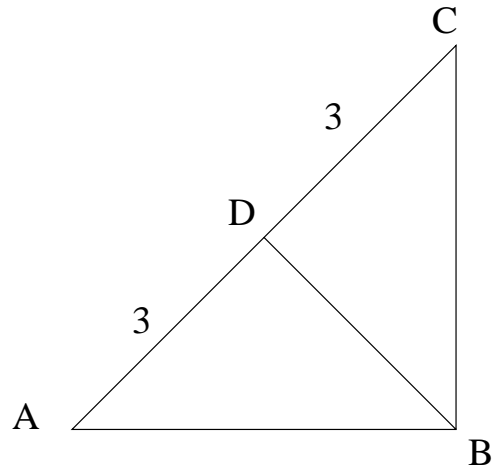


Fig. 1

- (a) 3    (b) 3.2  
 (c) 3.5    (d) 4  
 (e) None of the above

6. Suppose that  $f(x) = 1 - \frac{1}{(1-x)}$ . Find  $f(f(f(\dots f(4)\dots)))$ . Where there are 1997  $f$ 's in this composition.

- (a) 4    (b) 4/3  
 (c) 3/4    (d) 1/4  
 (e) None of the above

7. A  $33\frac{1}{3}$  r.p.m. stereo record has a spiral groove starting 6 inches from the center and ending 3 inches from the center. If it took approximately 18 minutes to play the record, about how long is the spiral groove? (Pick the closest number).

- (a) 1000 ft.                                      (b) 1400 ft.  
 (c) 1800 ft.                                      (d) 2200 ft.  
 (e) None of the above

8. A huge conical tank is to be made from a circular piece of sheet metal of radius 10 meters by cutting out a sector with vertex angle  $\theta$  and then welding the straight edges of the remaining piece together. Find the radius of the circular top of the resulting tank as a function of  $\theta$ .

- (a)  $5(\pi - \theta)/\pi$                       (b)  $5(\pi - 2\theta)/\pi$   
 (c)  $5(2\pi - \theta)/\pi$                       (d)  $5(2\pi - \theta)/2\pi$   
 (e) None of the above

9. Let  $S = \{10000, 10001, 10002, \dots, 99999\}$ , let

$$A = S \cap \{x : x \text{ has a digit which is a } 9\}.$$

Find the number of elements in  $A$ .

- (a) 37,512                                  (b) 42,256  
 (c) 43,006                                  (d) 52,488  
 (e) None of the above

10. Find the sum of all numbers between 0 and 300 which are multiples of 7 or 11.

- (a) 7,007                                      (b) 10,017  
 (c) 11,017                                      (d) 12,007  
 (e) None of the above

11. 10 women and 8 men are on a committee from which a subcommittee of size 7 is to be chosen. If at least 2 members of the subcommittee must be male, and if there must be more women than men on the subcommittee, how many ways can the subcommittee be chosen?

- (a) 18,012                                      (b) 18,024  
 (c) 18,056                                      (d) 18,816  
 (e) None of the above

12. If  $x = \sqrt[3]{4}$  and  $y = \sqrt[3]{8}$ , find  $xy$ .

- (a)  $1206/4455$                                   (b)  $1207/4456$   
 (c)  $1207/4455$                                   (d)  $1206/4456$   
 (e) None of the above

13. Which of the following statements below is a valid inference from the following assertions:

- i. For a dinosaur to be either purple or annoying it is necessary that he either sing or dance.
- ii. Barney (a famous dinosaur) does not dance.
- iii. Barney is purple.

- (a) Barney sings  
 (b) Barney is not annoying.  
 (c) Barney is annoying.  
 (d) Barney loves you.  
 (e) None of the above

14. If 5 adults and 2 children work together, a job can be done in a day. If only 2 adults work, then 6 children must work in order to complete the job in a day. The number of days that it takes for a child to do the job alone is:

- (a) 9    (b)  $25/3$   
 (c)  $26/3$     (d) 8  
 (e) None of the above

15. A parabola passes through the points  $(-1, 4)$ ,  $(1, 0)$ , and  $(2, 7)$ . Find the sum of the  $x$ -coordinates on the parabola associated with  $y = 64$ .

- (a)  $1/2$     (b)  $1/3$   
 (c)  $2/3$     (d) 1  
 (e) None of the above

16. How many integral roots does

$$x - \frac{7}{x-3} = 3 - \frac{7}{x-3}$$

have?

- (a) 0    (b) 1  
 (c) 2    (d) 3  
 (e) None of the above

17. If a bus travels an average of 40 m.p.h. between stops to travel  $m$  miles and it makes  $n$  stops of  $a$  minutes each, how long does the trip take?

- (a)  $3m + 2an$                                   (b)  $\frac{3m+2an}{12}$   
 (c)  $\frac{m+an}{40}$     (d)  $\frac{3m+2an}{120}$   
 (e) None of the above

18.  $\theta$  is an angle so that  $\cos(\theta)$  and  $\tan(\theta)$  have the same sign, and neither are 0. Find  $\frac{3|\sin(\theta + \pi)|}{\sin(\theta + \pi)}$ .

- (a) 3 (b) -3  
 (c) Can't be determined (d) -1  
 (e) None of the above

19. Male bees hatch from unfertilized eggs and so have a mother but no father. Female bees hatch from fertilized eggs. The number of ancestors that a male bee has in the 10th generation back is ?

- (a) 55 (b) 89  
 (c) 144 (d) 512  
 (e) None of the above

20. Suppose that  $f(x) = \frac{x+3}{x-1}$  and suppose that  $f(g(x)) = x$ . Find  $g(x)$ .

- (a)  $g(x) = \frac{x-1}{x+3}$   
 (b) Can't be determined from the given information  
 (c)  $g(x) = \frac{x-3}{x+1}$   
 (d)  $g(x) = \frac{x+3}{x-1}$   
 (e) None of the above

21. It takes 5 seconds for a clock to strike 6 o'clock beginning at 6:00 o'clock precisely. If the strikings are uniformly spaced, how long, in seconds, does it take to strike 12 o'clock?

- (a) 9.5 (b) 10  
 (c) 11 (d)  $14\frac{2}{3}$   
 (e) None of the above

22. Find the second largest solution of

$$2 \sin^2(n/2) + \cos^2(n) = 1$$

on  $[0, 2\pi]$ .

- (a)  $\frac{\pi}{3}$  (b)  $\pi$   
 (c)  $\frac{3\pi}{2}$  (d)  $\frac{5\pi}{6}$   
 (e) None of the above

23. Find the product of the g.c.d. and the l.c.m. of

$$\{12, 24, 72, 120, 288\}$$

- (a) 8640 (b) 17280  
 (c) 34560 (d) 11520  
 (e) None of the above

24. Suppose that triangle ABC has an obtuse angle at C and let D be the midpoint of side AC. Suppose E is on BC such that the segment DE is parallel to AB. What is true about the following three statements:

- i. E is the midpoint of BC.  
 ii. The length of DE is half the length of AB.  
 iii. DE bisects the altitude from AB to C.

- (a) Only i. is true  
 (b) Only i. and ii. are true  
 (c) All three are true  
 (d) Only i. and iii. are true  
 (e) None of the above

25. Consider the sequence  $8A + 2B, 6A + B, 4A, 2A - B, \dots$ . Which term of this sequence will have a coefficient of A which is twice the coefficient of B?

- (a) 49th (b) 10th  
 (c) 14th (d) 17th  
 (e) None of the above

26. Suppose that

$$x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}$$

then

- (a)  $x = 1$  (b)  $0 < x < 1$   
 (c)  $1 < x < 2$  (d)  $x$  is infinite  
 (e) None of the above

27. A college professor on vacation for  $n$  days observed that it rained 7 times, always in the morning or afternoon. When it rained in the afternoon, it was clear in the morning; there were five clear afternoons; there were six clear mornings. Find  $n$ .
- (a) 7 (b) 9  
(c) 10 (d) 11  
(e) None of the above
28. A sequence of equilateral triangles is drawn. The altitude of each is  $\sqrt{3}$  times the altitude of the preceding triangle, the difference between the areas of the first triangle and the sixth triangle is  $968\sqrt{3}$  square units. The perimeter of the first triangle is...
- (a) 12 (b) 18  
(c) 10 (d) 16  
(e) None of the above
29. Given two concentric circles with center  $P$ ,  $AC$  is a chord of the larger circle tangent to the smaller at  $B$ . If  $|BC| = 2$  units, the area of the annulus between the circles in square units is:
- (a)  $4\pi$  (b)  $2\pi$   
(c)  $8\pi$  (d)  $16\pi$   
(e) None of the above
30. If each nerf has at least two squiggles and each squiggle is on at least two nerfs then if there is at least one squiggle, the smallest number of nerfs possible is:
- (a) 1 (b) 2  
(c) 3 (d) 4  
(e) None of the above
31. Suppose  $x$  is a positive acute angle whose tangent is  $1/2$ . What is  $\sin(x)$ ?
- (a)  $\frac{1}{\sqrt{5}}$  (b)  $\frac{2}{\sqrt{5}}$   
(c)  $\frac{1}{2}$  (d)  $\frac{\sqrt{5}}{2}$   
(e) None of the above
32. If  $A$  is a set which has 32 subsets,  $B$  is a set which has 64 subsets, and  $A \cup B$  has 256 subsets, then how many elements are in  $A \cap B$ ?
- (a) 31 (b) 3  
(c) 6 (d) 5  
(e) None of the above
33. A right circular cylinder is given. If the radius is increased by 25% then the volume will stay the same if the height is increased/decreased by what percent?
- (a) Increased by 64% (b) Decreased by 36%  
(c) Decreased by 25% (d) Increased by 25%  
(e) None of the above
34. A man has \$2.46 in pennies, nickels, dimes and quarters. If he has an equal number of coins of each kind, then the total number of coins he has is:
- (a) 4 (b) 6  
(c) 24 (d) 36  
(e) None of the above
35. How far from the origin is the line  $3x + 4y = 10$ ?
- (a) 1.5 units (b) 2 units  
(c) 2.5 units (d) 2.75 units  
(e) None of the above
36. Find the ratio of the area to the circumference of the circle which is circumscribed about an equilateral triangle of side length 1.
- (a)  $\frac{2}{\sqrt{3}}$  (b)  $\frac{4}{\sqrt{3}}$   
(c)  $\frac{1}{\sqrt{3}}$  (d)  $\frac{1}{2\sqrt{3}}$   
(e) None of the above
37. Suppose that  $w, x, y$ , and  $z$  are arbitrary real numbers. Which statement is not necessarily true:
- (a)  $\frac{w}{x} + \frac{y}{z} = \frac{wz+xy}{xz}$   
(b) If  $w = x$  and  $y = z$  then  $wy = xz$ .  
(c) If  $w + y = x + y$  then  $w = x$ .  
(d) If  $wx = yx$  then  $w = y$ .  
(e) None of the above

38. A jar contains 7 amoeba. The number of amoeba doubles every minute and it takes 40 minutes to fill the jar. How long (in minutes) does it take to fill  $1/2$  the jar?
- (a) 20  
 (b) 35  
 (c) It can't be determined from the given information  
 (d)  $40 - \frac{139}{7}$   
 (e) None of the above
39. A function  $f(x)$  is called even if  $f(-x) = f(x)$  for all  $x$  and it is called odd if  $f(-x) = -f(x)$  for all  $x$ . Which of the following are true statements:
- i) An even function times an odd function is even.  
 ii) An even function plus an even function is even.  
 iii) An odd function times an odd function is even.
- (a) i) and ii) only  
 (b) i) and iii) only  
 (c) ii) and iii) only  
 (d) All three are true.  
 (e) None of the above
40. The graph of  $\{(x, y) | 4x^2 + 4y^2 - 8x + 8y + 12 = 0\}$  in the real plane is:
- (a) A circle                      (b) A hyperbola  
 (c) A point                        (d) A parabola  
 (e) None of the above

### Bonus Questions

41. Consider the set

$$P = \{(x, y) | x \text{ and } y \text{ are real numbers and } y > 0\}.$$

For real numbers  $c$  and  $r$  let  $L_{c,r}$  be the set of points of the form

$$\{(x, y) \in P | (x - c)^2 + y^2 = r^2\}.$$

Define  $L_a$  to be the set of points of the form

$$\{(a, y) \in P | y \text{ is a real number}\}.$$

Define a blurb to be any of the sets  $L_{c,r}$  or  $L_a$ . Show that every pair of points in  $P$  determines a unique blurb.

42. Using the notation of the previous problem, classify all blurbs which go through the point  $(0,1)$  and don't intersect  $L_6$ .