1. Let $\mathcal{L}$ denote the line parallel to the line $x = \frac{4}{3}y + 8$ and passing through the point $P(16, 2)$. What is the y–coordinate of the y–intercept of $\mathcal{L}$?

(a) -10 (b) -8

(c) 8 (d) 15

(e) None of the above

2. Let $f(x)$ be a polynomial of degree 2 and suppose that $f(x+1) - f(x) = 6x + 8$ and $f(1) = 16$. What is $f(2)$?

(a) 27 (b) 28

(c) 29 (d) 30

(e) None of the above

3. Consider Figure 1. How many squares of any size are in this figure?

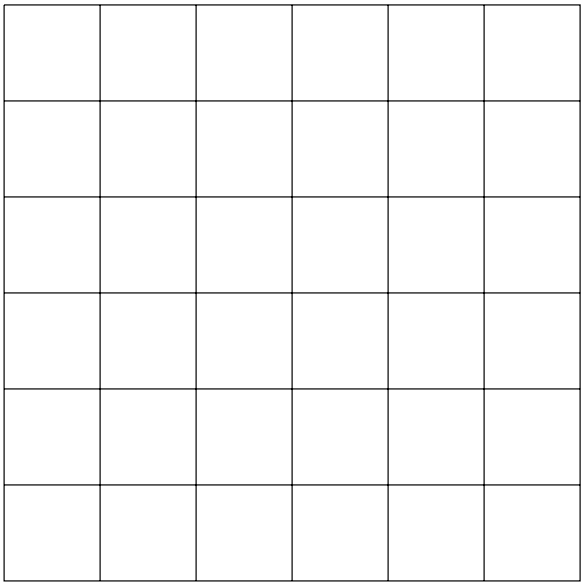

Fig. 1

(a) 89 (b) 90

(c) 91 (d) 92

(e) None of the above

4. Hannah, Mark and Tom have a combined age of 68. Hannah is twice as old as the difference between Mark and Tom's age. Mark and Tom's combined age is 16 times Hannah's age. Mark is older than Tom. How old is Tom?

(a) 31 (b) 32

(c) 33 (d) 34

(e) None of the above

5. Let $A, B \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Suppose that

$$7A3 + B25 = 1578.$$

What is $A + B$?

(a) 5 (b) 7

(c) 11 (d) 13

(e) None of the above

6. For which value of a does the equation

$$\frac{2x+5}{3x+8} = a$$

not have a solution in x?

(a) $-5/8$ (b) $5/8$

(c) $8/5$ (d) $2/3$

(e) None of the above

7. Given real numbers a and b, let $a \star b = a + b + ab$. Evaluate $3 \star (4 \star 5)$.

(a) 60 (b) 80

(c) 119 (d) 133

(e) None of the above

8. The Koffee Haus has 412 pounds of type A coffee, 208 pounds of type B coffee, and 800 pounds of guys with goatee beards and berets playing bongo drums, but I digress. The Haus makes two blends of coffee. The regular blend is 80% type A and 20% type B, while the special blend 50% type A and 50% type B. Let R and S denote the number of pounds of regular and special blend that the Haus should produce in order to exhaust their supplies of coffee. What is the greatest common divisor of R and S?

(a) 4 (b) 10

(c) 20 (d) 35

(e) None of the above

9. Find the sum of the squares of the solutions to $x^2 + 6x + 7 = 0$.

(a) 12 (b) 16

(c) 22 (d) 28

(e) None of the above

10. On a recent trip I averaged 20 miles per hour for 1 hour, 30 miles per hour for 3 hours and 60 miles per hour for 2 hours. Measured in miles per hour, which of the following is closest to the average speed for the entire trip?

(a) 36 (b) 37

(c) 38 (d) 39

(e) None of the above

11. When a 6 foot man stands 20 feet from a street light he casts a shadow of length 12 feet. Measured in feet, how tall is the streetlight?

(a) 15 (b) 16

(c) 17 (d) 18

(e) None of the above

12. Find the sum of the solutions to the equation

$$\sqrt{3x+4} = 2 + \sqrt{2x-4}$$

(a) 24 (b) 25

(c) 26 (d) 27

(e) None of the above

13. Three numbers are in proportions 2:3:5 and sum to 40. What is the second largest of these numbers?

(a) 8 (b) 9

(c) 10 (d) 11

(e) None of the above

14. On a recent trip I travelled 40% of the total distance at an average rate of 50 miles per hour. For the remainder of the trip I averaged 100 miles per hour. Which of the following is closest to the average speed for the entire trip?

(a) 67 (b) 71

(c) 80 (d) 83

(e) None of the above

15. Let I be the interval on which $|2x+3| \leq 4 + |x|$. What is the length of this interval?

(a) 6 (b) 7

(c) 8 (d) 9

(e) None of the above

16. How many solutions does the equation

$$1 + \sqrt{x+6} = \sqrt{2x+10}.$$

possess?

(a) 0 (b) 1

(c) 2 (d) 3

(e) None of the above

17. Let $f(x) = (2x+3)/(5x-1)$. It is well known that the inverse of f can be written in the form

$$f^{-1}(x) = \frac{x+b}{cx+d}.$$

Find $b+c+d$.

(a) 6 (b) 7

(c) 8 (d) 9

(e) None of the above

18. Find the coefficient of x^2 in the expansion of

$$(x+1)(2x-1)(3x+2)(4x-5).$$

(a) -39 (b) -38

(c) -37 (d) -36

(e) None of the above

19. Concerning Figure 2, what is the radius of the inscribed semicircle?

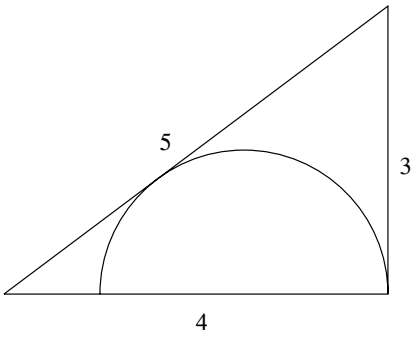

Fig. 2

(a) 5/8 (b) 3/2

(c) 11/8 (d) 5/3

(e) None of the above

20. The continued fraction

$$1+\cfrac{1}{x+\cfrac{2}{x+3}}$$

can be expressed as

$$\frac{x^2+ax+b}{x^2+cx+d}.$$

Find $a+b+c+d$.

(a) 11 (b) 12

(c) 13 (d) 14

(e) None of the above

21. On the island of Gnoman (that's right, Gnoman is an island) there is an unusual method of taxation. At the end of each year, the King's accountants take half of your gold and give you 6 pounds in return. Suppose that you start the year with 272 pounds of gold and each year you make 4 pounds of gold in tips and wages. How many pounds of gold will you have at the end of 8 years?

(a) 15 (b) 15.5

(c) 16 (d) 17

(e) None of the above

22. Let $\mathcal{L}$ denote the line which passes through the center of the circle

$$x^2+y^2+2x-8y+14=0$$

and the point $P(2,10)$. Which one of the following points is also on $\mathcal{L}$?

(a) (5,16) (b) (3,11)

(c) (6,20) (d) (4,13)

(e) None of the above

23. Suppose that $f(x+y)=f(x)f(y)$ for all real numbers x and y. If $f(1)=8$, what is $f(2/3)$?

(a) 1/8 (b) 2/3

(c) 4 (d) 8

(e) None of the above

24. $P(a,b)$ and $Q(c,d)$ are points on the parabola $y=x^2$. Suppose that the slope of the line through P and Q is 5 and that the x–coordinates of P and Q differ by 1. What is $b+d$?

(a) 5 (b) 13

(c) 25 (d) 41

(e) None of the above

25. Consider the triangle rendered in Figure 3. As luck would have it, $\sin(\alpha)$ can be expressed as a ratio a/b of integers in lowest terms. Find $a+b$.

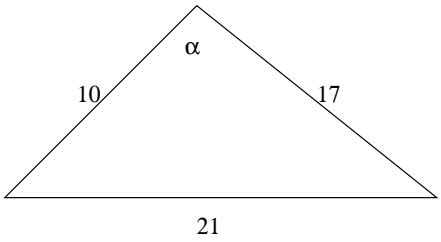

Fig. 3

(a) 113 (b) 137

(c) 157 (d) 169

(e) None of the above

26. Let $Q(x)$ denote the quotient which results from the division of $x^5+3x^4-x^3+8x^2-x+8$ by x^2+1. Find the sum of the squares of the coefficients of $Q(x)$.

(a) 36 (b) 37

(c) 38 (d) 39

(e) None of the above

27. Which of the following is equal to

$$\sqrt{11+2\sqrt{10}}+\sqrt{11-2\sqrt{10}}?$$

(a) 40 (b) 41

(c) 42 (d) 43

(e) None of the above

28. Suppose that $x+x^{-1}=3$. What is x^3+x^{-3}?

(a) 15 (b) 18

(c) 21 (d) 24

(e) None of the above

29. Let $f(x) = 1 + 1/(x+1)$. Which of the following is closest to $f(f(f(1)))$?

(a) 1.2 (b) 1.3

(c) 1.4 (d) 1.5

(e) None of the above

30. Consider the triangle pictured in Figure 4. Pass a line through each vertex parallel to the side opposite the given vertex. The three lines thus formed create a new triangle. What is the percent increase in the area of the new triangle over the old triangle?

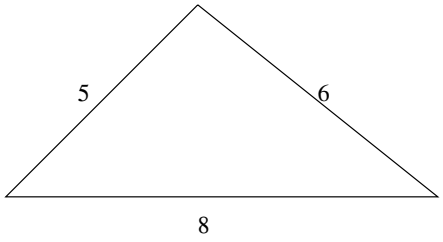

Fig. 4

(a) 200 (b) 300

(c) 400 (d) 500

(e) None of the above

31. Two rectangles pictured in Figure 5 intersect in a thirty degree angle. One of the rectangles is 2 inches by 20 inches and the other rectangle is 1 inch by 15 inches. What is the area of the union of these two rectangles?

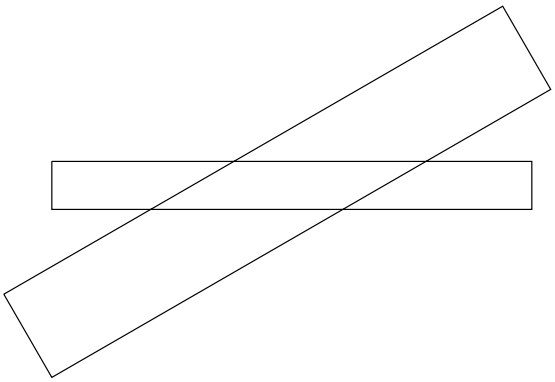

Fig. 5

(a) 49 (b) 50

(c) 51 (d) 52

(e) None of the above

32. The polynomial

$$P(x) = x^4 + 8x^3 + 23x^2 + 28x + 12$$

can be written in the form $P(x) = (x+2)^n Q(x)$, where n is a positive integer and $Q(x)$ is not divisible by $x+2$. What is n?

(a) 0 (b) 1

(c) 2 (d) 3

(e) None of the above

33. Concerning Figure 6, we have the following information: O is the center of the circle; AB is a diameter of the circle; $|AC| = 8$ and $|CB| = 2$; D is situated on the circle; and CD intersects AB at a right angle. What is $|CD|$?

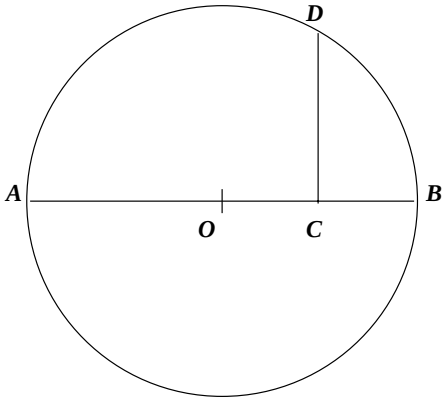

Fig. 6

(a) $2\sqrt{2}$ (b) 4

(c) $3\sqrt{2}$ (d) $\sqrt{10}$

(e) None of the above

34. Let $0 < \theta_1 < \theta_2 < \theta_3 < \cdots$ denote the positive solutions of the equation

$$3 + 3\cos(\theta) = 2\sin^2(\theta).$$

Then $\theta_3 + \theta_7$ can be expressed as $a\pi$, where a is an integer. What is a?

(a) 6 (b) 7

(c) 8 (d) 9

(e) None of the above

35. The natural domain of the function

$$f(x) = \frac{1}{\sqrt{3 - 2x - x^2}}$$

is an interval. What is the length of this interval?

(a) 3 (b) 3.5

(c) 4 (d) 4.5

(e) None of the above

36. Let f be the function which assigns to each point in the interval $(-\infty, -2]$ a number according to the rule $f(x) = x^2 + 4x + 5$. What is $f^{-1}(10)$?

(a) -4 (b) -3

(c) 1 (d) 5

(e) None of the above

37. Consider a triangle with vertices labelled A, B and C. If $|AB| = 30$, $|BC| = 26$, and $m(\angle BAC) = \pi/3$, what is $|AC|$?

(a) 17 (b) 18

(c) 19 (d) 20

(e) None of the above

38. Suppose that x is represented by the two digit number ab in base 8, and that $x = a^2 + b^2$, as well. What is b?

(a) 3 (b) 4

(c) 5 (d) 6

(e) None of the above

39. Consider a "chessboard" with 12 squares to a side. Remove every third column and every fourth row. How many squares remain?

(a) 56 (b) 60

(c) 72 (d) 80

(e) None of the above

40. In Figure 7, B and C are the centers of the circles; the circles are tangent at T; AT is a diameter of the circle centered at B; AS is tangent at S to the circle centered at C; and $\sin(\alpha) = 4/5$. Find $|TC|/|TB|$.

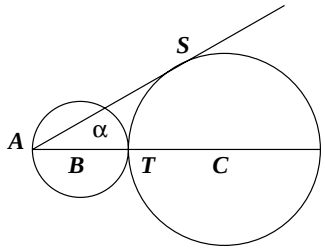

Fig. 7

(a) 5/4 (b) 8

(c) 4/5 (d) 6

(e) None of the above

Bonus Questions

41. How many distinct positive integer solutions are there for the equation

$$x^2 + 315 = y^2$$

42. Consult Figure 8. Given that the chords of the circle intersect at a right angle, find the radius of the circle.

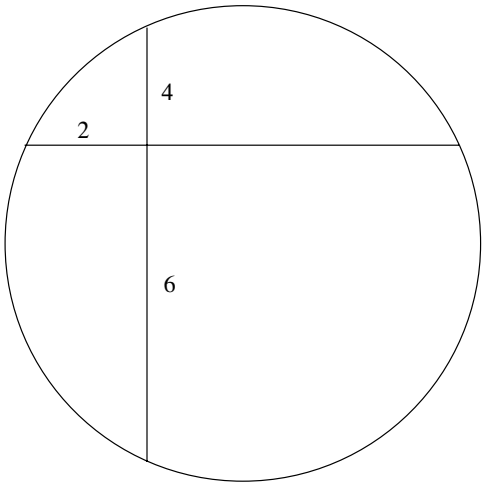

Fig. 8