The Conjugacy Question for Renewal Systems

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Abstract

A sofic shift space can be thought of as the set of bi-infinite walks on a labeled directed graph where the walker records the label on the edge he crosses. We call each walk a point and restrict our study to shifts arising from strongly connected graphs. These shifts are called irreducible and have the property that if u and v are finite blocks that appear in some points (possibly different), then there is another block w such that uwv appears in some point. We call a sofic shift a *renewal system* if it is given by a walk on a loop graph, namely a finite labeled directed graph with the additional property that each vertex, except possibly one, has a single incoming edge. We call a shift a *shift of finite type* if it can be represented by a graph and a labeling in such a way that every edge has a different label. These shifts are the building blocks of all sofic shifts in the sense that for each sofic shift there is an onto factor map from a shift of finite type to the sofic shift.

It has been shown that not every irreducible sofic shift is conjugate to a renewal system, but the question posed by Roy Adler, "Is every irreducible shift of finite type conjugate to a renewal system?" is still open. In this talk we introduce the ideas of a shift of finite type, a sofic shift, and a renewal system and give examples of each. We then discuss progress towards Adler's question.