Factoring Graphs as Direct Products

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Abstract

The *direct product* of two graphs G and H is the graph denoted $G \times H$ whose vertex set is the (set) Cartesian product $V(G) \times V(H)$. Two vertices (g_1, h_1) and (g_2, h_2) are adjacent in $G \times H$ if g_1 is adjacent to g_2 in G **and** h_1 and h_2 are adjacent in H. A *homomorphism* from graph G to graph H is a function $\varphi : V(G) \to V(H)$ such that if u and v are adjacent in G, then $\varphi(u)$ and $\varphi(v)$ are adjacent in H.

In this talk we will explore some of the more "algebraic" properties of the direct product. In particular, we will discuss the *prime factorization* of finite graphs with respect to direct products. We will consider the factorization of n-dimensional hypercubes with respect to the direct product.

Then we use ring theory (in particular, unique factorization domains) to prove a generalization of the following cancelation result of L. Lovász ["On the Cancellation Law Among Finite Relational Structures," *Periodica Mathematica Hungarica* Vol. 1 (2), 145–156 (1971)].

Theorem 1 *If A, B and C are graphs and if there is a homomorphism from A to C and a homomorphism from B to C, then $A \times C \simeq B \times C$ implies $A \simeq B$.*

This is joint work with Wilfried Imrich and Sandi Klavžar.