The achromatic and pseudoachromatic numbers of trees

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A **complete partition** of order k of a graph G = (V, E) is a partition $\Pi = \{V_1, V_2, \ldots, V_k\}$ of the vertex set V having the property that for any two sets V_i and V_j there is at least one edge between a vertex in V_i and a vertex in V_j. A **complete coloring** of a graph G is a complete partition in which every subset V_i in the partition has the property that no two vertices in V_i are joined by an edge. The largest order of a complete partition of a graph G is called the **pseudoachromatic number** of G and is denoted $\psi_s(G)$, while the largest order of a complete coloring of G is called the **achromatic number** and is denoted $\psi(G)$. By definition it follows that for any graph G, $\psi(G) \leq \psi_s(G)$. About 20 years ago S.T. Hedetniemi conjectured that for any tree T, $\psi(T) = \psi_s(T)$. This conjecture was shown to be false 10 years ago by Edwards, who found a tree with 408 vertices for which $\psi_s(T) = \psi(T) + 1$. We provide a much simpler counterexample with a tree having only 23 vertices and conjecture that not only is this a smallest possible counterexample, but that for all trees T, $\psi(T) \leq \psi_s(T) \leq \psi(T) + 1$.