

Experimental Mathematics and Exact Computation

Jonathan Michael Borwein, FRSC
Gordon M. Shrum Professor of Science & Director
Centre for Experimental and Constructive Mathematics
Simon Fraser University, Burnaby, V5A 1S6, BC

ABSTRACT.

The crucial role of high performance computing is now acknowledged throughout the physical, biological and engineering sciences: whether scientific computing, visualization, simulation or data-mining. The emergence of powerful mathematical computing environments, the growing availability of fast, seriously parallel computers and the pervasive presence of the internet allow for (pure and applied) mathematicians to partake of the same tools.

The unique features of our discipline make this both more problematic and more challenging. That said, many of the greatest computational benefits to mathematics are accessible through low-end “electronic blackboard” versions of experimental mathematics [1] and computing. Through a handful of examples, embracing both the high-end and the low-end, I intend to illustrate the opportunities and issues with which I am personally familiar. Many of the more sophisticated examples involve

$$\zeta(n) = \sum_{k=1}^{\infty} \frac{1}{k^n}$$

and its friends [2] and focus on the use of *Integer Relations Algorithms* [3,4] – recently described as among the ‘top ten’ algorithms of the century [5].

References

1. J.M. Borwein, P.B. Borwein, R. Girgensohn and S. Parnes, “Making Sense of Experimental Mathematics,” *Mathematical Intelligencer*, **18**, Number 4 (Fall 1996), 12–18. [CECM Preprint 95:032]¹
2. J.M. Borwein and D.M. Bradley, “Empirically determined Apéry-like formulae for zeta(4n+3),” *Experimental Mathematics*, **6** (1997), 181–194. [CECM Preprint 96:069]
3. Jonathan M. Borwein and Robert Corless, “Emerging tools for experimental mathematics,” *American Mathematical Monthly*, **106** (1999), 889–909. [CECM Preprint 98:110]
4. D.H. Bailey and J.M. Borwein, “Experimental Mathematics: Recent Developments and Future Outlook,” *World Mathematical Year 2000 Book*, Springer-Verlag, in press. [CECM Preprint 99:143]

¹All references are available at <http://www.cecm.sfu.ca/preprints/>.

5. J. Dongarra, F. Sullivan, The top 10 algorithms. *Computing in Science & Engineering* **2** (2000), 22–23. (See www.cecm.sfu.ca/personal/jborwein/algorithms.html)