§7.4–Arc Length

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Outline

Theorem (Arc length formula)

If f' is continuous on $[a, b]$, then the length of the curve $y = f(x)$, $a \leq x \leq b$, is

$$L = \int_a^b \sqrt{1 + (f'(x))^2} dx.$$

Problem

Find the length of the curve $y = x^{3/2}$, $0 \le x \le 44$.

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Solution

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Solution

- *Since* $y' = (3/2)x^{1/2}$, *hence* $(y')^2 = (9/4)x$, *it follows that*

$$L = \int_0^{44} \sqrt{1 + (9/4)x}\,dx = \int_1^{100} u^{1/2}\,du = \frac{1996}{3}.$$

Problem

Find the length of the curve $y = \frac{1}{10}x^5 + \frac{1}{6}x^{-3}$, $1 \leq x \leq 2$.

Solution

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- *We notice that* $y' = (1/2)x^4 - (1/2)x^{-4}$; *thus,* $(y')^2 = (1/4)x^8 - (1/2) + (1/4)x^{-8}$.

Solution

- *We notice that* $y' = (1/2)x^4 - (1/2)x^{-4}$; *thus,* $(y')^2 = (1/4)x^8 - (1/2) + (1/4)x^{-8}$.
- *Consequently,*

$$1 + (y')^2 = 1 + \left(\frac{1}{4}x^8 - \frac{1}{2} + \frac{1}{4}x^{-8}\right) = \frac{1}{4}x^8 + \frac{1}{2} + \frac{1}{4}x^{-8}$$

$$= \left(\frac{1}{2}x^4 + \frac{1}{2}x^{-4}\right)^2$$

Solution

- *We notice that* $y' = (1/2)x^4 - (1/2)x^{-4}$*; thus,* $(y')^2 = (1/4)x^8 - (1/2) + (1/4)x^{-8}$.
- *Consequently,*

$$1 + (y')^2 = 1 + \left(\frac{1}{4}x^8 - \frac{1}{2} + \frac{1}{4}x^{-8}\right) = \frac{1}{4}x^8 + \frac{1}{2} + \frac{1}{4}x^{-8}$$
$$= \left(\frac{1}{2}x^4 + \frac{1}{2}x^{-4}\right)^2$$

- *Finally*

$$L = \int_1^2 \left(\frac{1}{2}x^4 + \frac{1}{2}x^{-4}\right) dx =$$